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RL-TR-91-423, Vol I (of two)
Final Technical Report
December 1991

ALGORITHMS FOR SENSOR FUSION Decentralized Bayesian Hypothesis Testing with Feedback

Syracuse University

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RL-TR-91-423, Volume I (of two) has been reviewed and is approved for publication.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave Blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	December 1991	Final Oct 90 - May 91

4. TITLE AND SUBTITLE
ALGORITHMS FOR SENSOR FUSION, Decentralized Bayesian Hypothesis Testing with Feedback

5. FUNDING NUMBERS
C - F30602-89-C-0082
PE - 61102F
PR - 2305
TA - J8
WU - 04

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Syracuse University
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8. PERFORMING ORGANIZATION REPORT NUMBER
N/A

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Rome Laboratory (OCTS)
Griffiss AFB NY 13441-5700

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
RL-TR-91-423, Vol I (of two)

11. SUPPLEMENTARY NOTES (See reverse)
Rome Laboratory Project Engineer: Vincent C. Vannicola/OCTS/(315) 330-4437
*Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering in the Graduate School of Syracuse University

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)
Classicial signal detection involved centralized signal processing. A single sensor was employed for making observations which were processed centrally. The need for increased reliability and survivability of communication systems has led to the deployment of multiple sensors for signal detection. Various types of sensors are utilized to observe the environment. The collected data is sent to a central processor where classicial hypothesis testing procedures are employed for signal processing.

14. SUBJECT TERMS
Sensor Fusion, Signal Processing, Algorithms

15. NUMBER OF PAGES
154

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

Block 11 (Cont'd)

Prime Contractor: Kaman Sciences Corporation, 1500 Garden of the Gods Road, Colorado Springs CO 80933-7643

Acknowledgments

I would like to express my utmost gratitude and appreciation to my dissertation advisor, Professor Pramod Varshney, for his guidance and encouragement throughout the course of this work. I am very thankful to him for the constant support and helpful discussions.

I would like to thank Dr. R. Srinivasan for the helpful discussions during the early stages of this work. My thanks are also due to Dr. Vincent Vannicola for the many fruitful discussions.

Thanks are due to Professor Hari Krishna for serving as the reader and providing many constructive comments. Thanks are also due to Professors Carlos Hartmann, Harry Schwarzlander, Donald Weiner, and Can Isik for agreeing to be on my dissertation examining comittee.

I would like to express my sincere gratitude to the Electrical and Computer Engineering department for providing support and to Professor Norman Balabanian for his continued encouragement and interest in my work.

My parents have been a constant source of encouragement, inspiration, and untold sacrifices for which I will be for ever indebted.

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Chapter 1

Introduction

1.1 Background and Previous Work

Classical signal detection involved centralized signal processing. A single sensor was employed for making observations which were processed centrally. The need for increased reliability and survivability of communication systems has led to the deployment of multiple sensors for signal detection. Various types of sensors are utilized to observe the environment. The collected data is sent to a central processor where classical hypothesis testing procedures are employed for signal processing [1, 3]. Processing of observations is done only at the central processor. Hence, such communication systems are still centralized. The transmission of observations from the sensors requires communication channels with large communication bandwidth. Moreover, the computational load at the central processor increases unfavorably due to the increase in the number of observations to be processed. Naturally, the need for distributing the processing at the sensors was felt, thereby increasing the interest in the area of decentralized detection. Depending on the bandwidth constraints of the communication channels, some signal process-

ing is appropriately assigned to the peripheral sensors. These peripheral detectors perform some signal processing locally and transmit the results to a fusion center responsible for obtaining the final result.

The distributed detection system shown in Figure 1.1 has been considered quite extensively in the literature. The system consists of n local detectors observing the environment. Each local detector makes a decision concerning the hypothesis present based on its observations. Local detector decisions are then transmitted to the fusion center where they are combined to yield a global decision. The decentralized detection systems have been investigated using various approaches such as the Bayesian approach, the Neyman-Pearson approach, the min-max criterion and the Sequential Probability Ratio Test [4]-[16]. Tenney and Sandell [4] considered a distributed detection system with a fixed fusion center. They used the Bayesian approach to optimize a system consisting of two detectors with independent observations. Sadjadi [5] extended Tenney and Sandell's results to n detectors and M hypotheses. Chair and Varshney [6] used the Bayesian approach to optimize the fusion center with fixed local detectors. Hoballah and Varshney [7] presented a generalized Bayesian formulation of a decentralized detection system with a fusion center. Using the Person-By-Person-Optimal (PBPO) methodology, they derived the local detector and the fusion center decision rules. Reibman and Nolte [8] considered a decentralized detection system with non-Gaussian noise. In [9], Reibman and Nolte considered the general design and performance of several distributed detection system structures. Lauer and Sandell [10] used the Bayesian approach to optimize the distributed detection system with dependent observations at the local detectors. Ekchian and Tenney [11] optimized the tandem topology and various other system configurations. A simulation study of a specific decentralized detection system was conducted by Kushner and Pacut [12]. Teneketzis [13] developed a decentralized version of Wald's sequential detection problem. In addition, he considered the quickest detection problem in [14]. Srinivasan [15] considered the

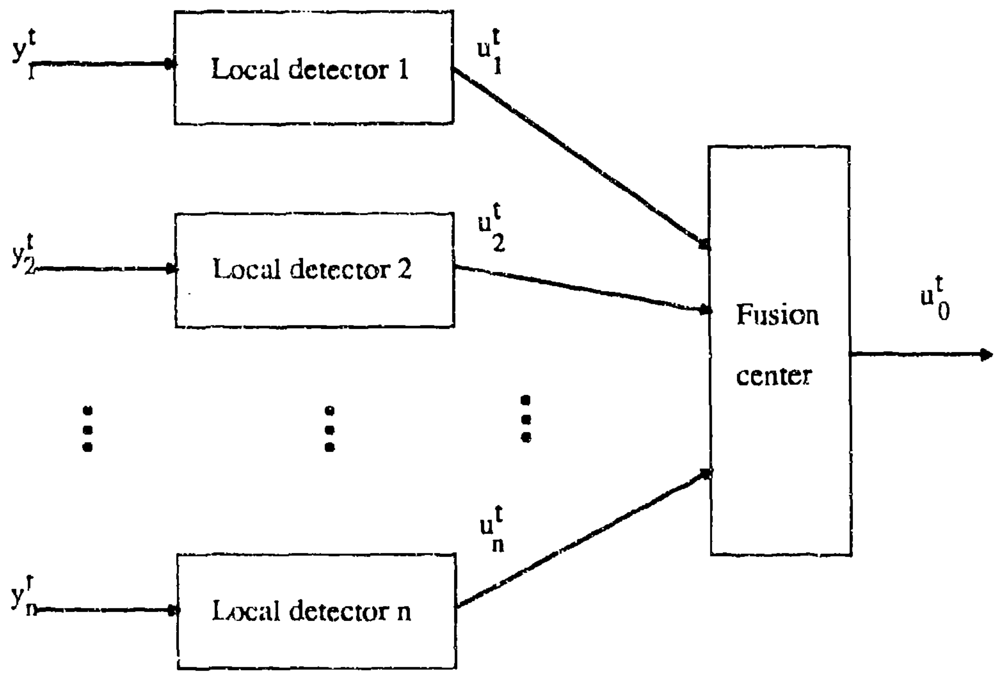

Fig. 1.1: A decentralized detection system

Neyman-Pearson approach for optimizing a decentralized detection system with a fixed fusion rule. Viswanathan and Thomopoulos [17] considered the two detector serial system and showed that it outperforms a parallel system with two detectors and a fusion center. Papastavrou and Athans [19] considered the tandem topology and derived asymptotic results for a serial system of n detectors. Tsitsiklis [21] discussed the advances in decentralized detection systems, computational issues and asymptotic results.

In most of the above work, information available to a local detector consisted of it's observations of the environment. Recently, Srinivasan [26] considered the availability of additional information such as the previous global decision at the local detectors. He used the Neyman-Pearson approach to optimize a decentralized detection system with feedback.

In this dissertation, we consider the decentralized detection system with feedback shown in Figure 1.2 and several variations from a Bayesian viewpoint. This system consists of n local detectors collecting observations from the environment. Each local detector makes a decision regarding the hypothesis present based on the collected observations and the previous global decision. These local decisions are transmitted to the fusion center where they are combined to yield a global decision. The global decision is transmitted back to all local detectors to aid them in their decision process. In addition to the study of the decentralized detection system with feedback, we will present a unified approach to the design and study of decentralized detection systems.

There are two major contributions of this dissertation. The first one is the demonstration of the fact that the performance of a decentralized detection system can be improved by the use of feedback. This improvement is achieved at the expense of increased communication. The other major contribution is a unified

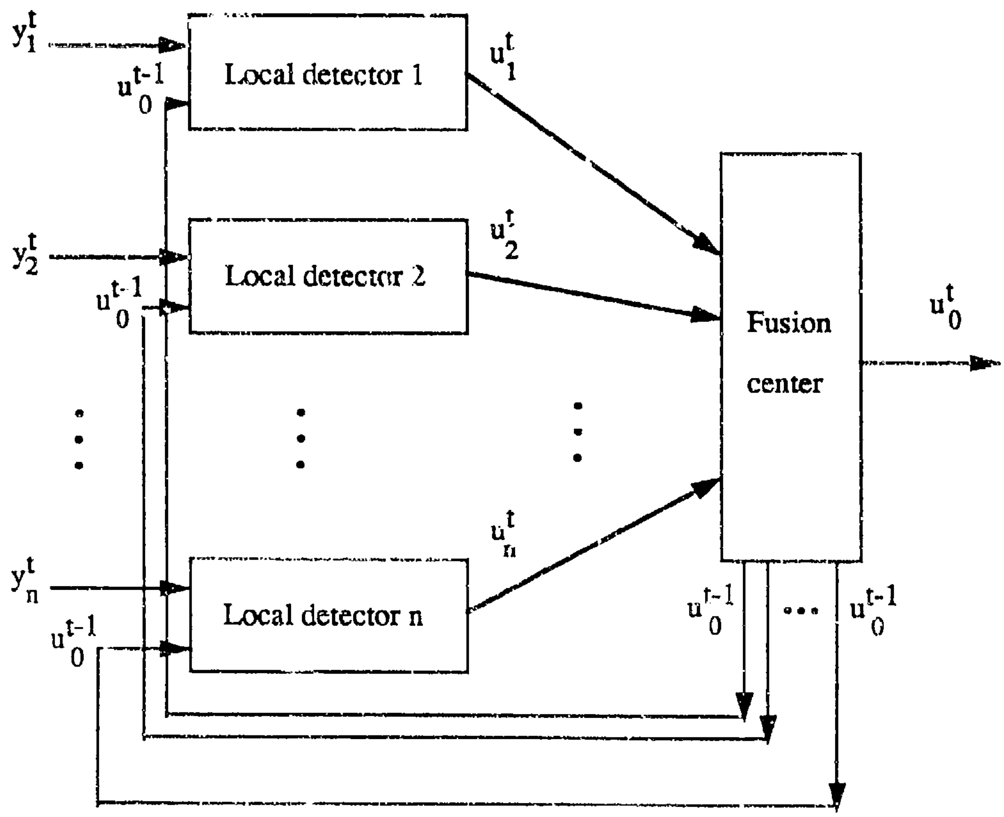

Fig. 1.2: A decentralized detection system with feedback.

representation of decentralized detection system with any topology along with an approach to obtain the PBPO decision rules at any detector of the decentralized detection system.

The general model for the decentralized detection problem consists of the following principal ingredients:

1. A set of random variables $\{H_i, \theta_j;\ i = 0, 1, \ldots,$ M-1; $j = 1, 2, \ldots, n\}$ that represent all the uncertainties in the problem and their distributions. The first variable represents the hypothesis, and is denoted by H_i, i=0,1,...,M-1. The other random variable is the noise present in the environment denoted by θ_j, j=1,2,...,n.

2. A set of observations Y=$\{y_1, y_2, \ldots, y_n\}$ which are given functions of the hypothesis present and the noise. In general, y_i, i=1,2,...,n, is a vector and is the observation available to the i^{th} decision maker (detector). From the given distribution of the noise θ, the conditional probability density function $p(y_i|H_j)$, j=0,1,...,M-1, is also known.

3. A set of decision variables U=$\{u_0, u_1, u_2, \ldots, u_n\}$ where each u_i represents the decision of the i^{th} decision maker. The decision u_i, i=0,1,2,...,n, is to take values appropriate to the decision space specified by the problem. In this formulation, u_0 is the global decision.

4. A set of decision rules $\Gamma = \{\gamma_0, \gamma_1, \gamma_2, \ldots, \gamma_n\}$, one for each decision maker (including the fusion center), where γ_i is a mapping from the observation space to the decision space, i.e.,

$$u_i = \gamma_i(y_i), \quad i = 1, 2, \ldots, n$$

and,

$$u_0 = \gamma_0(u_1, u_2, \ldots, u_n).$$

5. A cost (payoff) function $L(u_0, H_j)$ where u_0 is the final decision of the system and H_j is the hypothesis present.

The problem in decentralized detection systems is to

$$\text{Find } \gamma_i \text{ in } \Gamma \text{ , for all i such that}$$
$$E_{u_0,H_j}\{L(u_0, H_j)\} \text{ is minimized.}$$

In the next section, we present the dissertation organization.

1.2 Dissertation Organization

In this dissertation, we focus our attention on the decentralized detection system with feedback shown in Figure 1.2. In Chapter 2, we describe the decentralized detection system with feedback in detail and establish some initial results. Using the Person-By-Person-Optimal (PBPO) methodology from a Bayesian viewpoint, we derive the decision rules of the local detectors and the global decision maker. The optimum test at the local level is shown to be a likelihood ratio test for statistically independent observations at the local detectors. The number of observations is not assumed to be known a priori. Hence, optimization of this system is done for each time step t. In other words, it is assumed that the knowledge of the stopping time as to when the final decision is to be made is not available. The local threshold equation is a function of the previous global decision. The performance of the system is derived. In the remainder of the chapter, we assume that the stopping time of the decision process is known a priori, i.e. a known number of observations are available at each detector for processing. This is identified as the Fixed Sample Size (FSS) problem. Here, the PBPO solution methodology is again used and the system performance is optimized by taking into account the stopping time. We derive the PBPO decision rules both at the local detectors and at the fusion center for the fixed sample size problem. In addition, we consider a

detection system having a single detector with feedback. We derive the detector decision rule for the FSS problem. We establish the correspondence between the single detector system with feedback and the serial system thereby allowing us to utilize our results for the single detector system with feedback to serial networks. Furthermore, the results of the decentralized detection system with feedback are extended to more complex networks of serial nature. Thus, we provide a novel approach for the design and analysis of serial networks. Examples are presented throughout the chapter.

In Chapter 3, we consider a decentralized detection system with feedback and introduce memory at the local detectors, allowing them to store all previous observations. Using the Bayesian approach, we derive the PBPO solution for the decision rules. We show that the proposed system outperforms the conventional distributed detection system and the system without memory considered in Chapter 2 when more than one observation sample per detector are taken. Asymptotic results for this system are obtained and the probability of system error is shown to go to zero asymptotically. An important issue that arises in the decentralized detection system with feedback is that of data transmission. Due to the feedback links from the global decision maker to the local detectors , there is an increase in communication or data transmission. Two protocols are proposed and studied to achieve the desired reduction of data transmission. We show that the use of the proposed protocols reduces communication, on an average, to zero asymptotically. In other words, on an average no transmission of decisions is necessary among the system detectors asymptotically. An example is presented to illustrate the results obtained.

In Chapter 4, we consider the design of a decentralized detection system with an arbitrary topology. Inspired by Ho's definition of information structure [22], we define the communication structure of a system. Using this definition, decentral-

ized detection systems could be represented in terms of a communication matrix which shows the transmission paths of detector's decisions in a given system. We show the applicability of this definition to our study of the design of decentralized detection networks with arbitrary topologies. We generalize the definition of the communication matrix to enable us to study systems with feedback such as those in Chapters 2, and 3. Finally, using the PBPO solution methodology, we present a general approach for the derivation of decision rules for the FSS problem. We consider a number of examples and show that results available in the literature can be obtained using this general design approach. The generalized definition of the communication matrix and the general approach to the design of decision rules provide the necessary and sufficient tools for the study of decentralized detection systems with arbitrary configurations.

In Chapter 5, we present a summary and discuss the results obtained in this dissertation. Some directions for future research are also provided.

Chapter 2

The Bayesian Formulation of a Decentralized Detection System With Feedback

2.1 Introduction

The area of decentralized detection has been studied extensively in the literature recently. Decentralized detection systems have been proposed and investigated using various approaches such as the Bayesian approach, the Neyman-Pearson approach, the min-max criterion and the Wald's Sequential Probability Ratio Test [4]-[15]. Srinivasan [26] and Alhakeem et. al. [27] have recently investigated a decentralized detection system with feedback using the Neyman-Pearson approach. This was motivated by results such as [20] where it has been shown that improved channel capacity is achieved when a feedback link is employed. In this chapter, we study a decentralized detection system with feedback using the Bayesian formulation. In this system, the global decision at time step t is fed back to all local

detectors. Local detectors in turn operate on their observations as well as the received global decision to yield local decisions at time step t+1 which are then sent to the fusion center. A detailed description of this system is given in Section 2.2. In Section 2.3, we derive the decision rules at the local detectors and the fusion center using the PBPO solution methodology. The number of observations is not assumed to be known a priori and the optimization is done for each time step t. Probability of system error is derived. In Section 2.4, we consider the Fixed Sample Size problem (FSS) where we have an a priori knowledge of the stopping time t=T at which the final decision is made, i.e. the number of observations is known a priori. The system is optimized in such a manner that the system performance is optimum at the stopping time t=T. We formulate the FSS problem using the Bayesian approach and derive the global and local decision rules for any time t<T and t=T that minimize the average system cost at time t=T. In Section 2.5, we consider the single detector system with feedback and derive the decision rules for the FSS case using the Bayesian formulation. In Section 2.6, we show that the single detector system with feedback is equivalent to a serial system where the time step t is the same as the stage number n of the serial system. Hence, a decentralized detection system with feedback could be viewed as a serial system with n blocks in series where each block consists of local detectors and a fusion center. In Section 2.7, we discuss the results of this chapter. It is noted that even when the stopping time is known a priori, the decentralized detection system with feedback considered in this chapter cannot outperform the decentralized detection system without feedback that has been studied extensively in the literature. Numerical examples are presented throughout the chapter for illustration.

2.2 System Description and Problem Statement

We consider the binary hypothesis testing problem, with the two hypotheses denoted by H_0 and H_1 respectively, for the system shown in Figure 2.1. This system consists of n local detectors which communicate their decisions to a fusion center. At time step t, we denote the observation sample at the k^{th} detector by $y_k^t, k = 1, 2, \ldots, n$, and the local detector decision is denoted by $u_k^t, k = 1, 2, \ldots, n$. The global decision at time step t is denoted by u_0^t. The k^{th} detector takes an observation y_k^t at time step t, and based on its present observation and the previous global decision u_0^{t-1}, makes the local decision u_k^t and transmits it to the fusion center. The fusion center combines the incoming local decisions u_k^t, k=1,2, ..., n and generates the global decision u_0^t which is sent to all of the local detectors.

We assume that the joint conditional probability density functions $p(y_1^t, y_2^t, \ldots, y_n^t | H_i), i = 0, 1$ are known a priori. Each local detector uses a decision rule denoted by $\gamma_k^t(.)$ to make a decision u_k^t such that for k=1,2,...,n, we have the local decisions

$$u_k^t = \gamma_k^t(y_k^t, u_0^{t-1}).$$

Similarly, we denote the global decision rule by $\gamma_0^t(.)$ and the global decision is obtained as

$$u_0^t = \gamma_0^t(U^t)$$

where $U^t = (u_1^t, u_2^t, \ldots, u_n^t)$ is the vector of local detector decisions.
The problem is to find the PBPO decision rules $\gamma_k^t(.)$ for each detector k, k=0,1,2,...,n, so as to minimize the Bayesian cost function $J(\Gamma)$, where

$$\Gamma = \{\Gamma^t : t = 1, 2, \ldots\}.$$

Here, Γ^t is defined as:

$$\Gamma^t = \{\gamma_k^t(.) : k = 0, 1, \ldots, n\}$$

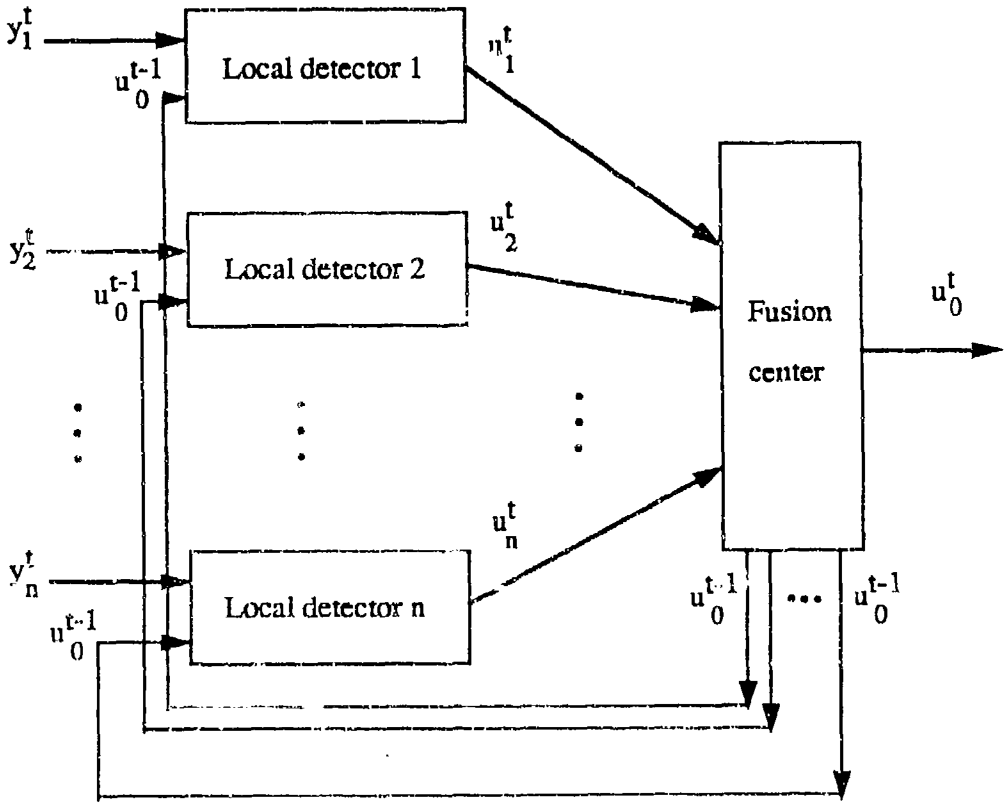

Fig. 2.1: A decentralized detection system with feedback.

The optimization of this system is carried out using the Person-By-Person optimization procedure for this team decision problem [25]. This system will be viewed as a team consisting of two members. One team member is the fusion center and the second team member is the aggregation of the individual detectors. The second team member can be further viewed as a team where individual detectors are assumed to be team members within their aggregate team. The equations resulting from the person-by-person optimization represent necessary conditions but not, in general, sufficient conditions to determine the globally optimal solution [25]. These equations are solved simultaneously to obtain the solution.

In the Bayesian approach, we assume the knowledge of the a priori probabilities $p(H_0)$ and $p(H_1)$. In addition, the cost of deciding $u_0 = H_i$ when the true hypothesis is H_j is denoted by C_{ij} i,j=0,1, and assumed to be known a priori. The Bayesian cost function to be minimized can be written as:

$$\begin{aligned} J(\Gamma^t) = \quad & C_{00}p(u_0^t = 0, H_0) + C_{01}p(u_0^t = 0, H_1) \\ & + C_{10}p(u_0^t = 1, H_0) + C_{11}p(u_0^t = 1, H_1) \end{aligned} \tag{2.1}$$

Denote the system probability of false alarm $p(u_0^t = 1|H_0)$ and the system probability of detection $p(u_0^t = 1|H_1)$ by $p_{f_0}^t$ and $p_{d_0}^t$ respectively. Rewriting (2.1) in terms of $p_{f_0}^t$ and $p_{d_0}^t$ we have:

$$J(\Gamma^t) = C_f p_{f_0}^t - C_d p_{d_0}^t + C \tag{2.2}$$

where

$$C_f = p(H_0)(C_{10} - C_{00})$$

$$C_d = p(H_1)(C_{01} - C_{11})$$

$$C = p(H_0)C_{00} + p(H_1)C_{01}$$

It is assumed that making a wrong decision is more costly than making a correct decision. This implies that C_f and C_d are greater then zero since $C_{10} > C_{00}$ and $C_{01} > C_{11}$. In the next section, we proceed with the system optimization and performance.

2.3 System Optimization and Performance

In this section, we utilize the PBPO solution methodology to minimize the Bayesian cost function in Equation (2.2). In Theorem 2.1 we derive the global decision rule $\gamma_0^t(.)$. The local decision rules are derived in Theorem 2.2. Before proceeding further, we assume that the observations at the local detectors are statistically independent. Therefore, the a priori knowledge of the conditional probability densities $p(y_1^t, y_2^t, \ldots, y_n^t|H_j)$ reduces to the a priori knowledge of the individual detector conditional probability densities $p(y_i^t|H_j)$, $i = 1, 2, \ldots, n$; j = 0,1. Theorem 2.1 is presented next.

THEOREM 2.1

For the decentralized detection system with feedback, the PBPO fusion rule for the Bayesian binary hypothesis testing problem is given by

$$\gamma_0^t(U^t) = u_0^t = \begin{cases} 1, & if\ \Lambda(U^t) > \dfrac{C_f}{C_d} \\ 0 & otherwise \end{cases} \tag{2.3}$$

where the likelihood ratio $\Lambda(U^t)$ is given by

$$\Lambda(U^t) = \frac{p(U^t|H_1)}{p(U^t|H_0)}$$

Proof:

Consider the cost function in Equation (2.2). We expand the probability of false alarm and the probability of detection around the decision vector U^t as follows,

$$J(\Gamma^t) = C_f \sum_{U^t} p(u_0^t = 1, U^t|H_0) - C_d \sum_{U^t} p(u_0^t = 1, U^t|H_1) + C.$$

Conditioning on U^t and expanding we get

$$J(\Gamma^t) = C_f \sum_{U^t} p(u_0^t = 1|U^t, H_0)p(U^t|H_0) - C_d \sum_{U^t} p(u_0^t = 1|U^t, H_1)p(U^t|H_1) + C.$$

Since u_0^t given U^t does not depend on the hypothesis present, we rewrite the previous expression as:

$$J(\Gamma^t) = \sum_{U^t} p(u_0^t = 1|U^t)[C_f p(U^t|H_0) - C_d p(U^t|H_1)] + C. \tag{2.4}$$

Due to the PBPO methodology being employed, we assume that the local detectors are fixed and minimize the cost function $J(\Gamma^t)$ by choosing the decision rule at the fusion center as

$$p(u_0^t = 1|U^t) = \begin{cases} 1, & if\ C_f p(U^t|H_0) - C_d p(U^t|H_1) < 0 \\ 0 & otherwise \end{cases}$$

which is the desired global decision rule $\gamma_0^t(U^t)$ given in (2.3).

Q.E.D.

THEOREM 2.2

The PBPO decision rule at the k^{th} local detector for the Bayesian binary hypothesis testing problem is given by

$$\gamma_k^t(y_k^t, u_0^{t-1}) = u_k^t = \begin{cases} 1, & if\ \dfrac{p(y_k^t|H_1)}{p(y_k^t|H_0)} > \eta_k^t(u_0^{t-1}) \\ 0 & otherwise \end{cases} \tag{2.5}$$

where $\eta_k^t(u_0^{t-1})$ is the k^{th} detector threshold at time step t defined as:

$$\eta_k^t(u_0^{t-1}) = \frac{C_f \sum_{U_k^t} f(U_k^t) p(U_k^t|u_0^{t-1}, H_0) p(u_0^{t-1}|H_0)}{C_d \sum_{U_k^t} f(U_k^t) p(U_k^t|u_0^{t-1}, H_1) p(u_0^{t-1}|H_1)} \tag{2.6}$$

and,

$$f(U_k^t) = p(u_0^t = 1|U_{k1}^t) - p(u_0^t = 1|U_{k0}^t)$$

$U_k^t : (u_1^t, u_2^t, \ldots, u_{k-1}^t, u_{k+1}^t, \ldots, u_n^t)$ the local detector decision vector U^t excluding the k^{th} detector decision.

$U_{ki}^t : (u_1^t, u_2^t, \ldots, u_k^t = i, \ldots, u_n^t)$ local detector decision vector U^t with the k^{th} detector decision u_k^t equal to i, i=0,1.

Proof:

We rewrite expression (2.4) explicitly in terms of the k^{th} local decision

$$
\begin{aligned}
J(\Gamma^t) = \sum_{U_k^t} \quad & p(u_0^t = 1|U_{k1}^t)[C_f p(U_{k1}^t|H_0) - C_d p(U_{k1}^t|H_1)] \\
& + p(u_0^t = 1|U_{k0}^t)[C_f p(U_{k0}^t|H_0) - C_d p(U_{k0}^t|H_1)] + C. \qquad (2.7)
\end{aligned}
$$

It should be observed that the summation above is over all the possiblities of the decision vector U_k^t. Substituting $p(U_{k0}^t|H_j) = p(U_k^t|H_j) - p(U_{k1}^t|H_j)$,j=0,1 in (2.7) and factoring out common terms, we have:

$$
\begin{aligned}
J(\Gamma^t) = \sum_{U_k^t} \quad & p(u_0^t = 1|U_{k1}^t)[C_f p(U_{k1}^t|H_0) - C_d p(U_{k1}^t|H_1)] \\
& - p(u_0^t = 1|U_{k0}^t)[C_f p(U_{k1}^t|H_0) - C_d p(U_{k1}^t|H_1)] \\
& + p(u_0^t = 1|U_{k0}^t)(C_f p(U_k^t|H_0) - C_d p(U_k^t|H_1)) + C.
\end{aligned}
$$

Factoring the terms in square brackets out,

$$
\begin{aligned}
J(\Gamma^t) = \sum_{U_k^t} \quad & [p(u_0^t = 1|U_{k1}^t) - p(u_0^t = 1|U_{k0}^t)] \\
& \times [C_f p(U_{k1}^t|H_0) - C_d p(U_{k1}^t|H_1)] \\
& + p(u_0^t = 1|U_{k0}^t)(C_f p(U_k^t|H_0) - C_d p(U_k^t|H_1)) + C. \qquad (2.8)
\end{aligned}
$$

Observing that the last two terms are not involved in the optimization of the k^{th} local detector due to the PBPO procedure being employed, we drop those terms in the subsequent equations and denote the new cost function by $J^1(\Gamma^t)$. Next, we expand $J^1(\Gamma^t)$ in u_0^{t-1}, the previous global decision, and $Y^t = (y_1^t, y_2^t, \ldots, y_n^t)$ the observation vector of local detectors:

$$
\begin{aligned}
J^1(\Gamma^t) = \sum_{U_k^t} \quad & [p(u_0^t = 1|U_{k1}^t) - p(u_0^t = 1|U_{k0}^t)] \\
& \times \int_{Y^t} \sum_{u_0^{t-1}} [C_f p(U_{k1}^t, u_0^{t-1}, Y^t|H_0) \\
& - C_d p(U_{k1}^t, u_0^{t-1}, Y^t|H_1)]. \qquad (2.9)
\end{aligned}
$$

For notational convenience, we assume that the integration is over the appropriate variables indicated with the integral sign $\int$ and the term dY^t will not be written explicitly. This convention is followed throughout the dissertation. Letting $p(u_0^t = 1|U_{k1}^t) - p(u_0^t = 1|U_{k0}^t) = f(U_k^t)$, and expanding (2.9) by conditioning on u_0^{t-1} and Y^t, we have,

$$J^1(\Gamma^t) = \sum_{U_k^t} f(U_k^t) \int_{Y^t} \sum_{u_0^{t-1}} [C_f p(U_{k1}^t|u_0^{t-1}, Y^t, H_0) p(u_0^{t-1}, Y^t|H_0) - C_d p(U_{k1}^t|u_0^{t-1}, Y^t, H_1) p(u_0^{t-1}, Y^t|H_1)]. \quad (2.10)$$

The local decision vector U_{k1}^t given both the previous global decision and the observation vector Y^t does not depend on the hypothesis H_j, j=0,1. In addition, assuming the observations are independent in time, the previous global decision u_0^{t-1} is independent of the observation vector Y^t. We rewrite (2.10) as:

$$J^1(\Gamma^t) = \sum_{U_k^t} f(U_k^t) \int_{Y^t} \sum_{u_0^{t-1}} [C_f p(U_{k1}^t|u_0^{t-1}, Y^t) p(u_0^{t-1}|H_0) p(Y^t|H_0) - C_d p(U_{k1}^t|u_0^{t-1}, Y^t) p(u_0^{t-1}|H_1) p(Y^t|H_1)]. \quad (2.11)$$

Moreover, assuming that the observations at detectors i and j, i≠j, are independent of each other, Equation (2.11) is rewritten as,

$$\begin{aligned} J^1(\Gamma^t) = \sum_{U_k^t} f(U_k^t) \int_{Y^t} \sum_{u_0^{t-1}} & [C_f p(u_0^{t-1}|H_0) p(U_{k1}^t|u_0^{t-1}, y_k^t) p(y_k^t|H_0) \\ & \prod_{i=1, i\neq k}^{n} p(u_i^t|u_0^{t-1}, y_i^t) p(y_i^t|H_0) \\ -C_d p(u_0^{t-1}|H_1) \; & p(U_{k1}^t|u_0^{t-1}, y_k^t) p(y_k^t|H_1) \\ & \prod_{i=1, i\neq k}^{n} p(u_i^t|u_0^{t-1}, y_i^t) p(y_i^t|H_1)]. \end{aligned} \quad (2.12)$$

Factoring out the term $p(U_{k1}^t|u_0^{t-1}, y_k^t)$, we have

$$\begin{aligned} J^1(\Gamma^t) = \sum_{U_k^t} & \sum_{u_0^{t-1}} f(U_k^t) \int_{y_k^t} p(U_{k1}^t|u_0^{t-1}, y_k^t) \\ & \int_{Y_k^t} [C_f p(u_0^{t-1}|H_0) p(y_k^t|H_0) \textstyle\prod_{i=1, i\neq k}^{n} p(u_i^t|u_0^{t-1}, y_i^t) p(y_i^t|H_0) \\ & - C_d p(u_0^{t-1}|H_1) p(y_k^t|H_1) \textstyle\prod_{i=1, i\neq k}^{n} p(u_i^t|u_0^{t-1}, y_i^t) p(y_i^t|H_1)] \end{aligned} \quad (2.13)$$

where $Y_k^t : (y_1^t, y_2^t, \ldots, y_{k-1}^t, y_{k+1}^t, \ldots, y_n^t)$ the observation vector Y^t excluding the k^{th} detector observation.

Integrating over Y_k^t and rearranging (2.13) we have,

$$
\begin{aligned}
J^1(\Gamma^t) = \sum_{u_0^{t-1}} \quad & \int_{y_k^t} p(U_{k1}^t | u_0^{t-1}, y_k^t) \textstyle\sum_{U_k^t} f(U_k^t) \\
& [C_f p(u_0^{t-1}|H_0) p(U_k^t|u_0^{t-1}, H_0) p(y_k^t|H_0) \\
& - C_d p(u_0^{t-1}|H_1) p(U_k^t|u_0^{t-1}, H_1) p(y_k^t|H_1)]. \qquad (2.14)
\end{aligned}
$$

To minimize the cost function given by Equation (2.14) we choose

$$
p(U_{k1}^t | u_0^{t-1}, y_k^t) = \begin{cases} 1 & if\ A_1 > A_0 \\ 0 & otherwise \end{cases} \qquad (2.15)
$$

where

$$
A_1 = \sum_{U_k^t} f(U_k^t) C_d p(y_k^t|H_1) p(u_0^{t-1}|H_1) p(U_k^t|u_0^{t-1}, H_1)
$$

$$
A_0 = \sum_{U_k^t} f(U_k^t) C_f p(y_k^t|H_0) p(u_0^{t-1}|H_0) p(U_k^t|u_0^{t-1}, H_0)
$$

The k^{th} detector decision rule therefore is given by rewriting (2.15) as:

$$
\gamma_k^t(y_k^t, u_0^{t-1}) = u_k^t = \begin{cases} 1, & if\ \dfrac{p(y_k^t|H_1)}{p(y_k^t|H_0)} > \eta_k^t(u_0^{t-1}) \\ 0 & otherwise \end{cases} \qquad (2.16)
$$

where $\eta_k^t(u_0^{t-1})$ is the k^{th} detector threshold at time step t defined as:

$$
\eta_k^t(u_0^{t-1}) = \frac{C_f \sum_{U_k^t} f(U_k^t) p(U_k^t|u_0^{t-1}, H_0) p(u_0^{t-1}|H_0)}{C_d \sum_{U_k^t} f(U_k^t) p(U_k^t|u_0^{t-1}, H_1) p(u_0^{t-1}|H_1)} \qquad (2.17)
$$

as stated in Equation (2.6).

Q.E.D.

At time step t=1, there is no feedback. At this step, the fusion rule has the same form as given in Theorem 2.1. However, the local decision rules are single

threshold likelihood ratio tests given by

$$\gamma_k^1(y_k^1) = u_k^1 = \begin{cases} 1, & if\ \dfrac{p(y_k^1|H_1)}{p(y_k^1|H_0)} > \eta_k^1 \\ 0 & otherwise \end{cases} \tag{2.18}$$

where η_k^1 is the k^{th} detector threshold at time step 1 defined as:

$$\eta_k^1 = \frac{C_f \sum_{U_k^1} f(U_k^t) p(U_k^1|H_0)}{C_d \sum_{U_k^1} f(U_k^t) p(U_k^1|H_1)} \tag{2.19}$$

$$f(U_k^t) = p(u_0^1 = 1|U_k^1 = 1) - p(u_0^1 = 1|U_k^1 = 0).$$

From Theorems 2.1 and 2.2, the following observations regarding the decentralized detection system with feedback can be made:

- The optimum test at a local detector is a likelihood ratio test for statistically independent observations.
- The k^{th} local detector threshold $\eta_k^t(u_0^{t-1})$ is a function of the previous global decision u_0^{t-1} as given in Equation (2.6). For the binary hypothesis testing problem, two thresholds exist since the previous global decision takes two values. The threshold $\eta_k^t(u_0^{t-1})$ is also a function of the probabilities of system false alarm and miss at the previous step, namely $p_{f_0}^{t-1}$ and $p_{m_0}^{t-1}$
- At every time step t, there are 2^n fusion rule equations and $2n$ local threshold equations to be solved for the binary hypothesis testing problem.
- Since the local detector's thresholds change from one time step to the next, the optimum fusion rule changes as well.

System Performance

Next, we consider the performance of the decentralized detection system with

feedback. In the general case, the performance is described in terms of the Bayes cost $J(\Gamma)$ given in Equation (2.2). Here, however, we consider the special case of minimum probability of error criterion, i.e. $C_{00} = C_{11} = 0$ and $C_{01} = C_{10} = 1$, and characterize the system performance in terms of the system probability of error denoted by $p^t_{e_0}$. The system probability of error $p^t_{e_0}$ is given by

$$p^t_{e_0} = p^t_{f_0} p(H_0) + p^t_{m_0} p(H_1) \tag{2.20}$$

where $p^t_{m_0}$ is the system probability of miss. We expand $p^t_{f_0}$ defined as $p(u^t_0 = 1 | H_0)$ in terms of u^{t-1}_0,

$$\begin{aligned} p^t_{f_0} = \quad & p(u^t_0 = 1 | u^{t-1}_0 = 1, H_0) p(u^{t-1}_0 = 1 | H_0) \\ & + p(u^t_0 = 1 | u^{t-1}_0 = 0, H_0) p(u^{t-1}_0 = 0 | H_0). \end{aligned} \tag{2.21}$$

Replacing $p(u^{t-1}_0 = 0 | H_0)$ by $1 - p(u^{t-1}_0 = 1 | H_0)$ and rearranging terms, we have

$$\begin{aligned} p^t_{f_0} = \quad & p(u^{t-1}_0 = 1 | H_0)[p(u^t_0 = 1 | u^{t-1}_0 = 1, H_0) \\ & - p(u^t_0 = 1 | u^{t-1}_0 = 0, H_0)] + p(u^t_0 = 1 | u^{t-1}_0 = 0, H_0). \end{aligned}$$

This may be rewritten as:

$$p^t_{f_0} = p^{t-1}_{f_0}[p^t_{f_0}(u^{t-1}_0 = 1) - p^t_{f_0}(u^{t-1}_0 = 0)] + p^t_{f_0}(u^{t-1}_0 = 0) \tag{2.22}$$

where

$$p^t_{f_0}(u^{t-1}_0 = i) = p(u^t_0 = 1 | u^{t-1}_0 = i, H_0).$$

Introducing the local decision vector U^t in the above expression, we have

$$\begin{aligned} p^t_{f_0}(u^{t-1}_0 = i) = \quad & p(u^t_0 = 1 | u^{t-1}_0 = i, H_0) = \\ & \textstyle\sum_{U^t} p(u^t_0 = 1 | U^t, u^{t-1}_0, H_0) p(U^t | u^{t-1}_0 = i, H_0). \end{aligned} \tag{2.23}$$

Observing that the global decision u^t_0 conditioned on U^t does not depend on u^{t-1}_0 and H_0, Equation (2.23) yields

$$p^t_{f_0}(u^{t-1}_0 = i) = \sum_{U^t} p(u^t_0 = 1 | U^t) p(U^t | u^{t-1}_0 = i, H_0). \tag{2.24}$$

Similarly, the probability of system miss $p^t_{m_0}$ is written as:

$$p^t_{m_0} = p^{t-1}_{m_0}[p^t_{m_0}(u^{t-1}_0 = 0) - p^t_{m_0}(u^{t-1}_0 = 1)] + p^t_{m_0}(u^{t-1}_0 = 1) \tag{2.25}$$

where the probability of system miss $p^t_{m_0}(u^{t-1}_0 = i)$ is expressed as:

$$\begin{aligned} p^t_{m_0}(u^{t-1}_0 = i) &= p(u^t_0 = 0|u^{t-1}_0 = i, H_1) \\ &= \sum_{U^t} p(u^t_0 = 0|U^t)p(U^t|u^{t-1}_0 = i, H_1). \end{aligned} \tag{2.26}$$

Substituting Equations (2.22) and (2.25) in (2.20), we obtain the probability of system error $p^t_{e_0}$. At time step t=1 the system probability of error is given by

$$p^1_{e_0} = p^1_{f_0}p(H_0) + p^1_{m_0}p(H_1) \tag{2.27}$$

where

$$\begin{aligned} p^1_{f_0} &= \sum_{U^1} p(u^1_0 = 1|U^1)p(U^1|H_0) \\ p^1_{m_0} &= \sum_{U^1} p(u^1_0 = 0|U^1)p(U^1|H_1). \end{aligned}$$

Next, we consider an example where some numerical results are obtained.

Example 2.1

We consider a system consisting of two local detectors and a fusion center. The binary hypothesis testing problem is considered. Under both hypotheses, the input observations at each detector are assumed to have a Rayleigh distribution. For simplicity, the signal-to-noise ratio (SNR) at the two detectors is assumed to be equal and is denoted by ϵ. As shown by DiFranco and Rubin [28], for this model, the probability of false alarm and the probability of detection are given by

$$p^t_{f_k} = ((1+\epsilon) \times \eta^t_k)^{-1-(1/\epsilon)}$$

and,

$$p^t_{d_k} = (p^t_{f_k})^{1/(1+\epsilon)}.$$

The above equations in addition to Equations (2.19), and (2.27) are used to evaluate the system probability of error and thresholds at time t=1. The results are then used in Equations (2.6), (2.20), (2.22),(2.25) to obtain the system probability of error and thresholds for t>1. The minimum probability of error criterion is assumed, i.e. , $C_{00} = C_{11} = 0$ and $C_{01} = C_{10} = 1$. Also, the a priori probabilities are assumed to be equal. In this example, we consider two fusion rules namely the OR and the AND fusion rules. For the OR fusion rule, we plot the threshold values $\eta_k^t(u_0^{t-1} = 0)$ and $\eta_k^t(u_0^{t-1} = 1)$ vs. SNR for different values of t in Figures 2.2 and 2.3 respectively. The probability of system error $p_{e_0}^t$ vs. SNR for different values of t is plotted in Figure 2.4. Similarly, for the AND fusion rule, we plot the threshold values $\eta_k^t(u_0^{t-1} = 0)$ and $\eta_k^t(u_0^{t-1} = 1)$ vs. SNR in Figures 2.5 and 2.6 respectively. The probability of system error $p_{e_0}^t$ vs. SNR is plotted in Figure 2.7.

The plot in Figure 2.2 shows that the threshold $\eta_k^t(u_0^{t-1} = 0)$ increases as a function of time and as a function of SNR. The plot in Figure 2.3 shows that the threshold $\eta_k^t(u_0^{t-1} = 1)$ decreases as a function of time and as a function of SNR. As SNR goes to infinity, the threshold $\eta_k^t(u_0^{t-1} = 1)$ goes to zero and $\eta_k^t(u_0^{t-1} = 0)$ goes to infinity . The plot in Figure 2.4 shows that the probability of system error $p_{e_0}^t$ decreases as a function of time and as a function of SNR as expected. It can be observed that $p_{e_0}^t$ goes to zero as SNR value increases to infinity and as time step t goes to infinity . For the AND rule, the thresholds and the probability of system error shown in Figures 2.5, 2.6 and 2.7 follow a similar behavior. It should be noted that Figure 2.6 shows that the threshold values given $u_0^{t-1} = 1$ are independent of the time parameter t due to the use of the AND fusion rule.

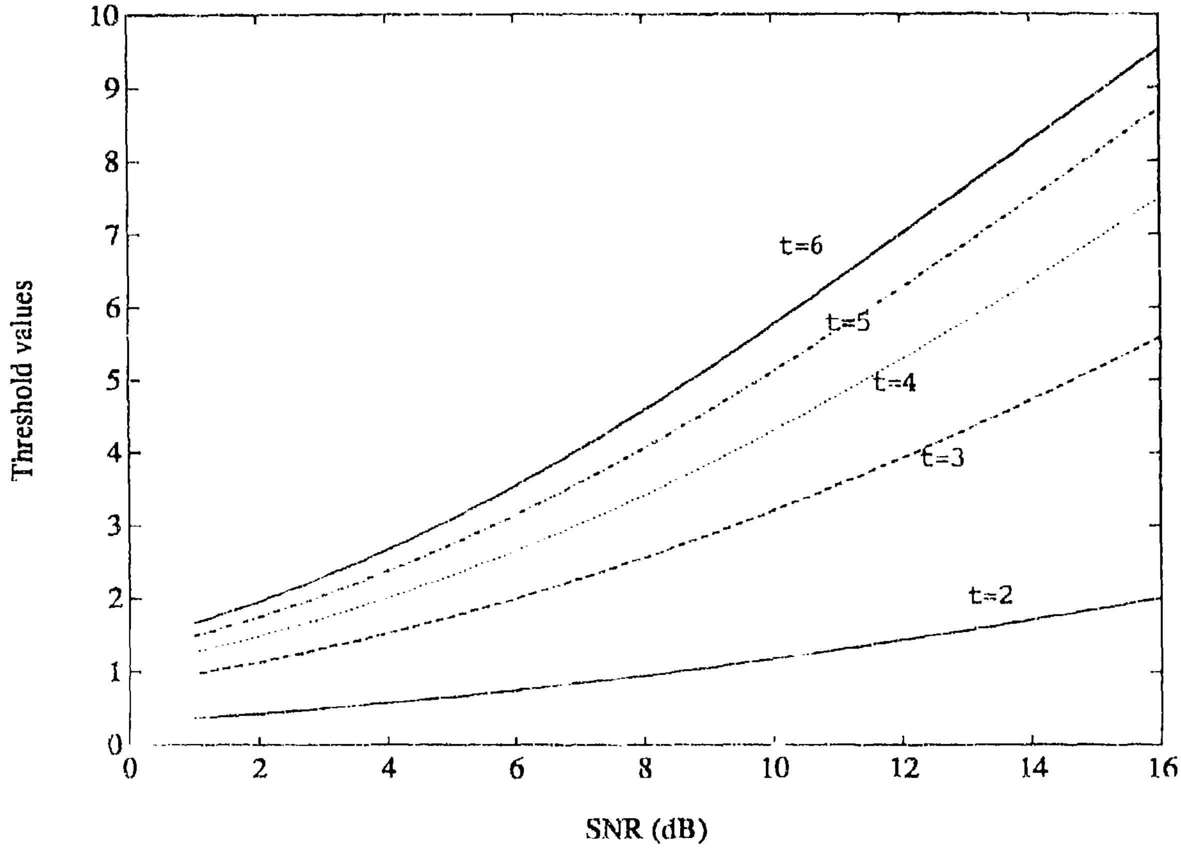

Figure 2.2: Threshold values given that u_0^{t-1}=0, OR fusion rule

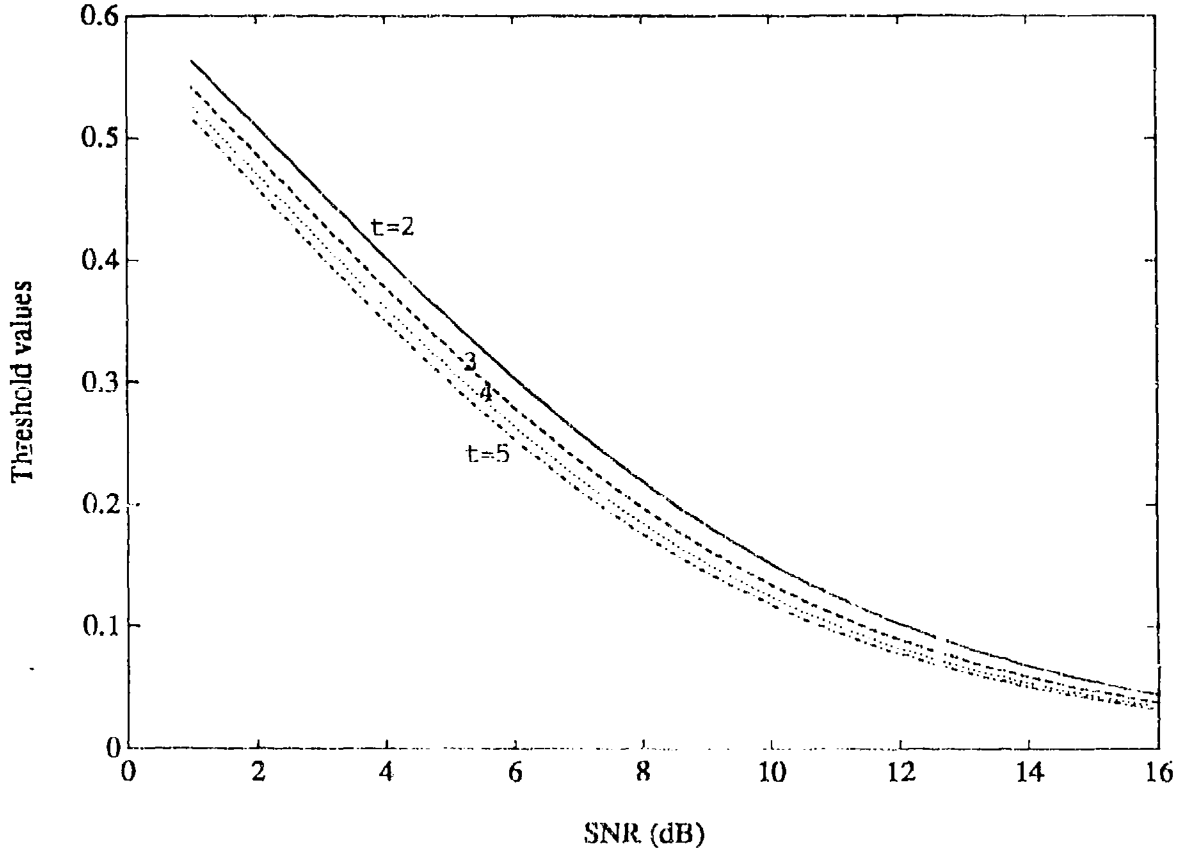

Figure 2.3: Threshold values given that u_0^{t-1}=1, OR fusion rule

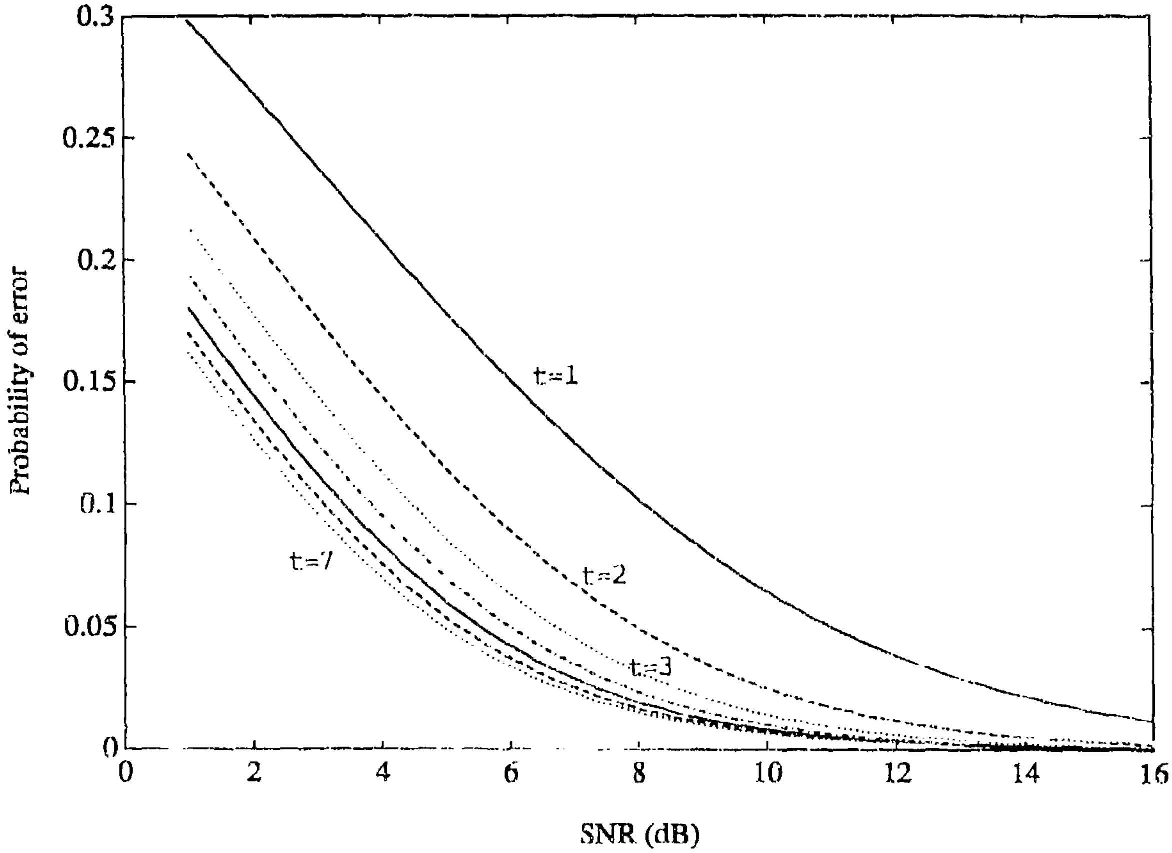

Figure 2.4: The system probability of error, OR fusion rule

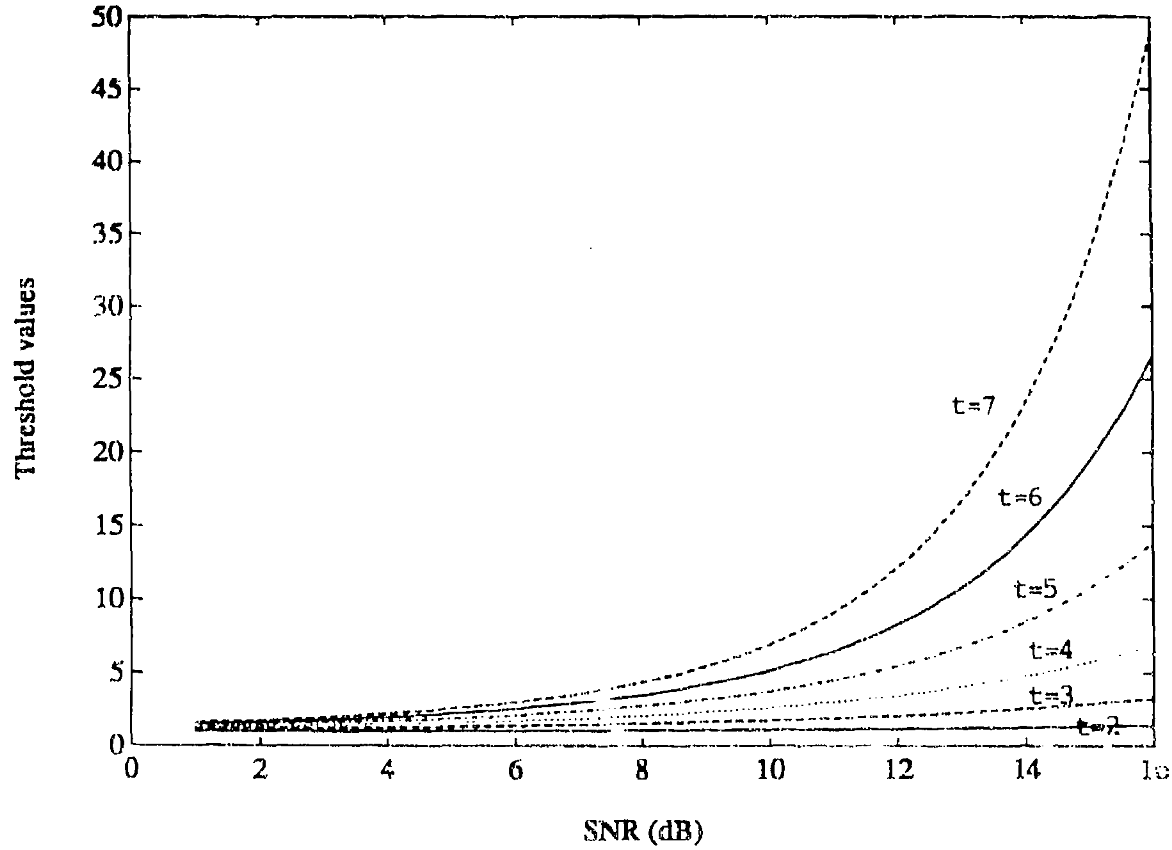

Figure 2.5: Threshold values given that u_0^{t-1}=0, AND fusion rule

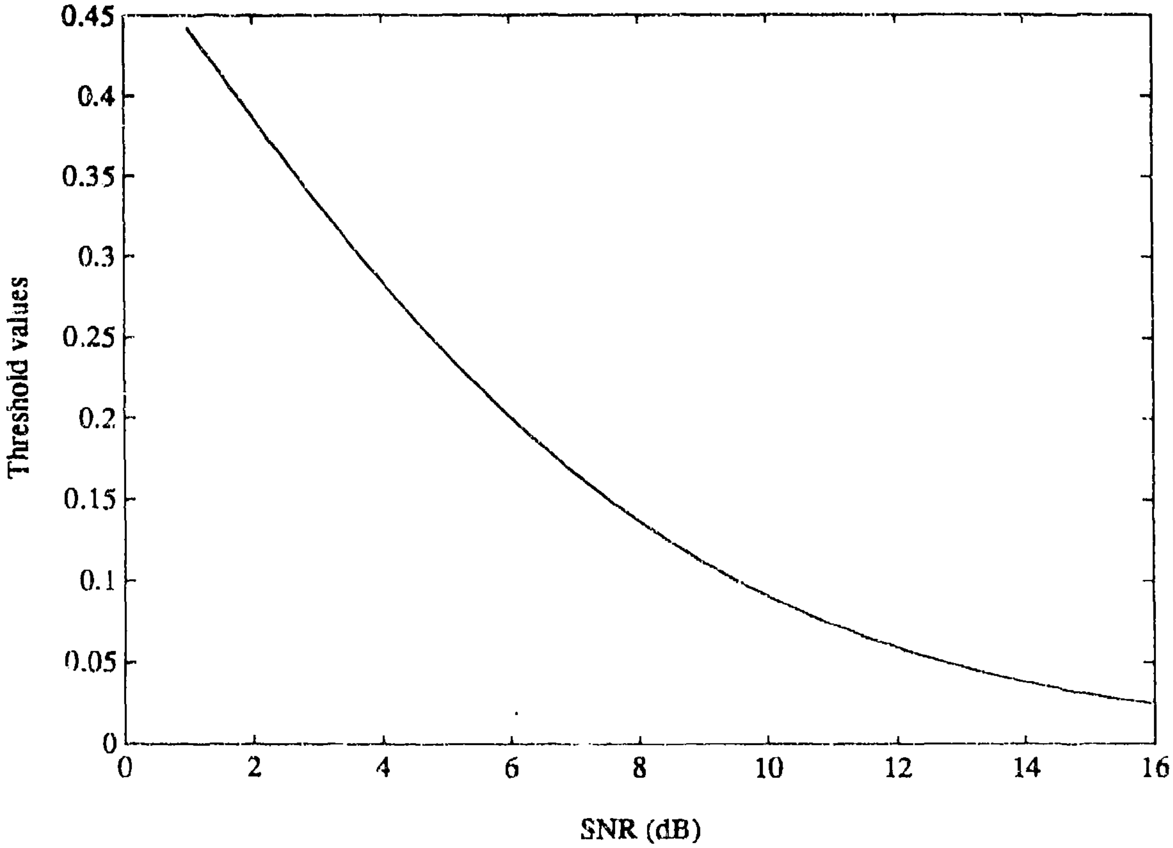

Figure 2.6: Threshold values given that u_0^{t-1}=1, AND fusion rule

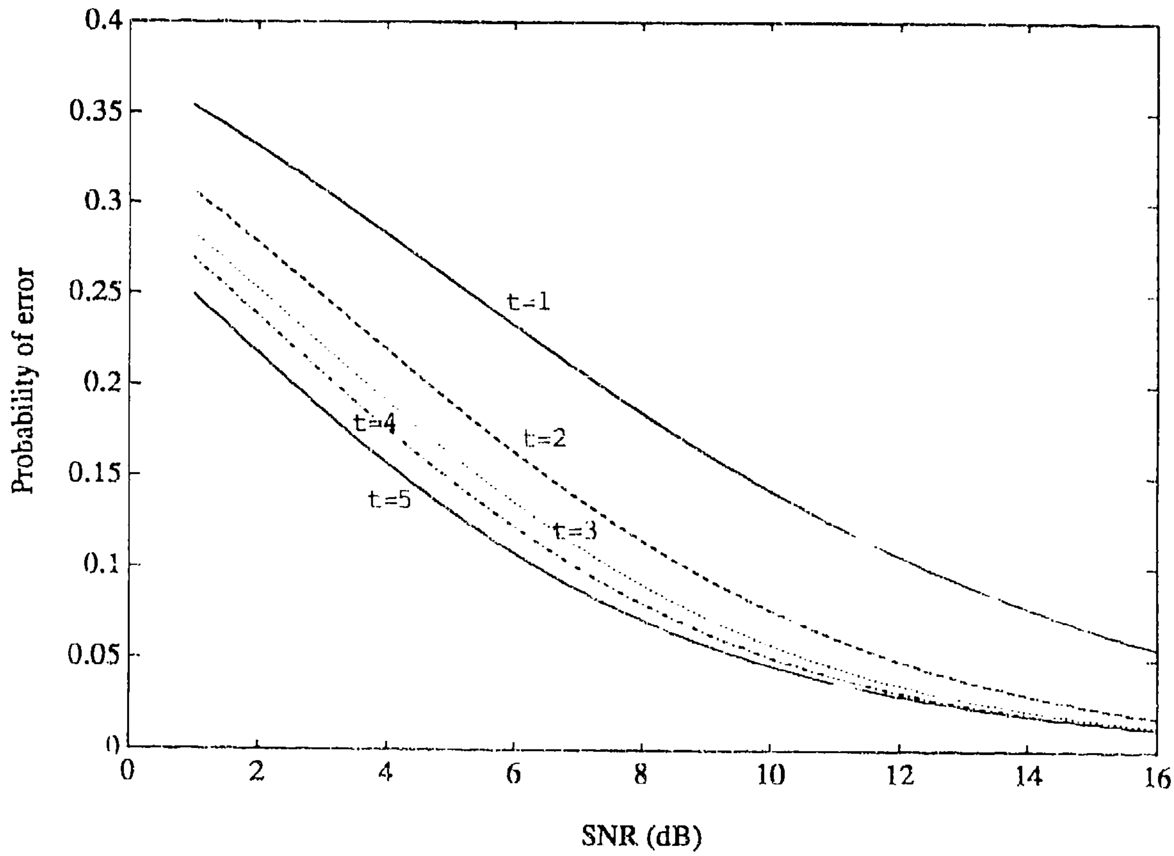

Figure 2.7: The system probability of error, AND fusion rule

2.4 The Fixed Sample Size Problem

In the previous section, we considered the decentralized detection system with feedback. The stopping time was not known a priori and the system was optimized at each time step. In this section, we consider the case where the stopping time T at which the final global decision is to be made is known a priori. We refer to this problem as the Fixed Sample Size (FSS) problem. Next, we define the FSS problem in more detail and design the system so as to minimize the Bayesian cost function using the PBPO solution methodology.

Problem Statement and System Optimization

We again consider the binary hypothesis testing problem for the system shown in Figure 2.1. The system operation is the same as before. In the FSS problem, the stopping time t=T is known a priori. Hence, the problem is to find the optimal decision rules γ_k^t for each detector k=0, 1, 2, ..., n so as to minimize the Bayesian cost function $J(\Gamma)$, where

$$\Gamma = \{\Gamma^t : t = 1, 2, ..., T\} \tag{2.28}$$

and as before

$$\Gamma^t = \{\Gamma_k^t(.) : k = 0, 1, ..., n\}.$$

Observe that the minimization is over the entire set of decision rules up to time step t=T. We assume that the conditional probability density functions $p(y_1^t, y_2^t, ..., y_n^t | H_j)$, j=0, 1 are known a priori. In addition, the probabilities $p(H_0)$, $p(H_1)$ and the costs C_{ij} =cost { decide $u_0^T = H_i |$ hypothesis present $= H_j$}, i, j=0, 1 are all assumed to be known a priori. We assume that the observations at the k^{th} detector are independent in time (temporal independence). In adddition, the observations at the k^{th} detector are assumed to be independent from those at the r^{th} detector, $r \neq k$, (spatial independence). Hence, the a priori knowledge of the conditional density $p(y_1^t, y_2^t, ..., y_n^t | H_j)$ reduces to the a priori knowledge of the

individual detector conditional probability densities $p(y_i^t|H_j)$, i=1, 2, ..., n; j=0, 1.

The Bayesian cost function $J(\Gamma)$ to be minimized is written as:

$$\begin{aligned} J(\Gamma) = \quad & C_{00}p(u_0^T = 0, H_0) + C_{01}p(u_0^T = 0, H_1) \\ & + C_{10}p(u_0^T = 1, H_0) + C_{11}p(u_0^T = 1, H_1) \end{aligned} \tag{2.29}$$

which reduces to

$$J(\Gamma) = C_f p_{f_0}^T - C_d p_{d_0}^T + C \tag{2.30}$$

where C_f, C_d, and C are as defined in Section 2.2.

We should observe the effect of fixing the total number of observations available in (2.30) where the probabilities of system miss and false alarm $p_{m_0}^T$, $p_{f_0}^T$ are a function of the final time step t=T. We would like to find the set of decision rules Γ such that the Bayesian cost function $J(\Gamma)$ of (2.30) is minimized. In Theorem 2.3, we derive the global decision rules $\gamma_0^t(.)$ at any time $t \leq T$. The local decision rules are derived in Theorem 2.4.

THEOREM 2.3

For the decentralized detection system with feedback shown in Figure 2.1, the PBPO fusion rule for the Bayesian binary hypothesis testing problem with a fixed sample size is given by

$$\gamma_0^T(U^T) = u_0^T = \begin{cases} 1 & if\ \dfrac{p(U^T|H_1)}{p(U^T|H_0)} > \dfrac{C_f}{C_d} \\ 0 & otherwise \end{cases} \tag{2.31}$$

and for t<T

$$\gamma_0^t(U^t) = u_0^t = \begin{cases} 1 & if\ \Lambda(U^t) > \dfrac{C_f g(t,0)}{C_d g(t,1)} \\ 0 & otherwise. \end{cases} \tag{2.32}$$

where $g(t,j) = p(u_0^T = 1|u_0^t = 1, H_j) - p(u_0^T = 1|u_0^t = 0, H_j)$

Proof:

We consider the cost function $J(\Gamma)$ given in Equation (2.30). We expand the probability of false alarm and detection around the local decision vector U^T at time step t=T. Hence,

$$\begin{aligned} J(\Gamma) = \quad & C_f \textstyle\sum_{U^T} p(u_0^T = 1, U^T|H_0) \\ & C_d \textstyle\sum_{U^T} p(u_0^T = 1, U^T|H_1) + C \end{aligned} \tag{2.33}$$

Conditioning on U^T and expanding, we get

$$\begin{aligned} J(\Gamma) = \quad & C_f \textstyle\sum_{U^T} p(u_0^T = 1|U^T, H_0)p(U^T|H_0) \\ & -C_d \textstyle\sum_{U^T} p(u_0^T = 1|U^T, H_1)p(U^T|H_1) + C \end{aligned} \tag{2.34}$$

The global decision u_0^T given the decision vector U^T does not depened on the hypothesis present; we rewrite (2.34) as

$$J(\Gamma) = \sum_{U^T} p(u_0^T = 1|U^T)[C_f p(U^T|H_0) - C_d p(U^T|H_1)] + C \tag{2.35}$$

Since C is fixed, we minimize the cost function $J(\Gamma)$ of (2.34) by using the following descision rule

$$p(u_0^T = 1|U^T) = \begin{cases} 1 & if\ C_f p(U^T|H_0) - C_d p(U^T|H_1) < 0 \\ 0 & otherwise \end{cases}$$

which is the desired decision rule $\gamma_0^T(U^T)$ at time step T as given in (2.31). The global decision rule $\gamma_0^t(U^t)$ for t<T is derived by expanding the cost $J(\Gamma)$ of (2.30) around the global decision u_0^t and the local decision vector U^t. In this case, we have

$$\begin{aligned} J(\Gamma) = \quad & C_f \textstyle\sum_{u_0^t,U^t} p(u_0^T = 1, u_0^t, U^t|H_0) \\ & -C_d \textstyle\sum_{u_0^t,U^t} p(u_0^T = 1, u_0^t, U^t|H_1) + C \end{aligned} \tag{2.36}$$

Conditioning on u_0^t and U^t and expanding, we have

$$
\begin{aligned}
J(\Gamma) = \quad & C_f \textstyle\sum_{u_0^t,U^t} p(u_0^T = 1|u_0^t, U^t, H_0)p(u_0^t|U^t, H_0)p(U^t|H_0) \\
& -C_d \textstyle\sum_{u_0^t,U^t} p(u_0^T = 1|u_0^t, U^t, H_1)p(u_0^t|U^t, H_1)p(U^t|H_1) + C
\end{aligned}
\tag{2.37}
$$

The global decision u_0^T given the global decision u_0^t does not depend on the local decision vector U^t since u_0^t is a function of U^t. Moreover, the global decision u_0^t given the local decision vector U^t does not depend on the hypothesis present. Therefore, the cost function $J(\Gamma)$ of (2.37) reduces to

$$
\begin{aligned}
J(\Gamma) = C_f \sum_{u_0^t,U^t} \quad & p(u_0^T = 1|u_0^t, H_0)p(u_0^t|U^t)p(U^t|H_0) \\
& -C_d \textstyle\sum_{u_0^t,U^t} p(u_0^T = 1|u_0^t, H_1)p(u_0^t|U^t)p(U^t|H_1) + C
\end{aligned}
\tag{2.38}
$$

Expanding the above explicitly in terms of the two possibilities of the global decision u_0^t, we get

$$
\begin{aligned}
J(\Gamma) = \sum_{U^t} \quad & C_f p(u_0^T = 1|u_0^t = 1, H_0)p(u_0^t = 1|U^t)p(U^t|H_0) \\
& -C_d p(u_0^T = 1|u_0^t = 1, H_1)p(u_0^t = 1|U^t)p(U^t|H_1) \\
& +C_f p(u_0^T = 1|u_0^t = 0, H_0)p(u_0^t = 0|U^t)p(U^t|H_0) \\
& -C_d p(u_0^T = 1|u_0^t = 0, H_1)p(u_0^t = 0|U^t)p(U^t|H_1) + C
\end{aligned}
\tag{2.39}
$$

Substituting $p(u_0^t = 0|U^t)$ by $1 - p(u_0^t = 1|U^t)$ and factoring out common terms in $J(\Gamma)$ of (2.39), we have

$$
\begin{aligned}
J(\Gamma) = \sum_{U^t} \quad & [C_f p(u_0^T = 1|u_0^t = 1, H_0) - C_f p(u_0^T = 1|u_0^t = 0, H_0)]p(U^t/H_0) \\
& \times p(u_0^t = 1|U^t) \\
& -C_d[p(u_0^T = 1|u_0^t = 1, H_1) - p(u_0^T = 1|u_0^t = 0, H_1)]p(U^t|H_1) \\
& \times p(u_0^t = 1|U^t) \\
& +C_f p(u_0^T = 1|u_0^t = 0, H_0)p(U^t|H_0) - C_d p(u_0^T = 1|u_0^t = 0, H_1) \\
& \times p(U^t|H_1) + C
\end{aligned}
\tag{2.40}
$$

The last three terms are independent of the optimization of the global decision rule at time step t. Therefore, we drop these terms in the subsequent analysis and denote the new cost function by $J^1(\Gamma)$. Factoring out the common term in (2.40), the cost function $J^1(\Gamma)$ is written as:

$$\begin{aligned} J^1(\Gamma) = \sum_{U^t} \quad & p(u_0^t = 1|U^t)[C_f p(U^t|H_0)[p(u_0^T = 1|u_0^t = 1, H_0) \\ & -p(u_0^T = 1|u_0^t = 0, H_0)] - C_d p(U^t|H_1) \\ & \times[p(u_0^T = 1|u_0^t = 1, H_1) - p(u_0^T = 1|u_0^t = 0, H_1)]] \qquad (2.41) \end{aligned}$$

Letting $p(u_0^T = 1|u_0^t = 1, H_j) - p(u_0^T = 1|u_0^t = 0, H_j) = g(t,j)$ in (2.41), we have

$$\begin{aligned} J^1(\Gamma) = \sum_{U^t} p(u_0^t = 1|U^t) \quad & [C_f p(U^t|H_0) g(t,0) \\ & - C_d p(U^t|H_1) g(t,1)] \qquad (2.42) \end{aligned}$$

To minimize the cost function $J^1(\Gamma)$ in (2.42) we choose

$$p(u_0^T = 1|U^t) = \begin{cases} 1 & if\ A < 0 \\ 0 & otherwise \end{cases} \qquad (2.43)$$

where

$$A = C_f p(U^t|H_0) g(t,0) - C_d p(U^t|H_1) g(t,1)$$

With a little rearrangement, Equation (2.43) becomes the global decision rule $\gamma_0^t(U^t)$ at any time step t<T as given in Equation (2.32) of Theorem 2.3.

Q.E.D.

It should be noted that the above global decision rules for t<T were derived with the objective of optimizing the system performance at time t=T. On the other hand, the global decision rules of the previous section were derived with the objective of optimizing the system performance at time t independently of the future decisions, i.e. the objective was to optimize the performance of the decisions at every time t which may not result in optimal performance for the

decision at time t=T. Next, we present the derivations of the local decision rules $\gamma_k^t(.)$, k=1,2,...,n.

THEOREM 2.4

For the decentralized detection system with feedback shown in Figure 2.1, the PBPO local decision rules for the Bayesian binary hypothesis testing problem with a fixed sample size is given by

$$u_k^t = \gamma_k^t(y_k^t, u_0^{t-1}) = \begin{cases} 1 & if\ \Lambda(y_k^t) > \eta_k^t(u_0^{t-1}) \\ 0 & otherwise \end{cases} \tag{2.44}$$

where $\eta_k^t(u_0^{t-1})$ is the k^{th} detector threshold given for t=T as

$$\eta_k^T(u_0^{T-1}) = \frac{C_f \sum_{U_k^T} f(U_k^T) p(U_k^T, u_0^{T-1}|H_0)}{C_d \sum_{U_k^T} f(U_k^T) p(U_k^T, u_0^{T-1}|H_1)} \tag{2.45}$$

$$f(U_k^T) = p(u_0^T = 1|U_{k1}^T) - p(u_0^T = 0|U_{k0}^T)$$

and for t<T

$$\eta_k^t(u_0^{t-1}) = \frac{C_f g(t,0) \sum_{U_k^t} f(U_k^t) p(U_k^t, u_0^{t-1}|H_0)}{C_d g(t,1) \sum_{U_k^t} f(U_k^t) p(U_k^t, u_0^{t-1}|H_1)} \tag{2.46}$$

$$g(t,j) = p(u_0^T = 1|u_0^t = 1, H_j) - p(u_0^T = 1|u_0^t = 0, H_j).$$

Proof:

We first derive the local decision rules at time step t=T. We recall Equation (2.35) and write it explicitly in terms of the k^{th} local decision

$$\begin{aligned} J(\Gamma) = \quad & \textstyle\sum_{U_k^T} p(u_0^T = 1|U_{k1}^T)[C_f p(U_{k1}^T|H_0) - C_d p(U_{k1}^T|H_1)] \\ & + \textstyle\sum_{U_k^T} p(u_0^T = 1|U_{k0}^T)[C_f p(U_{k0}^T|H_0) - C_d p(U_{k0}^T|H_1)] + C \end{aligned} \tag{2.47}$$

where

$$U_{ki}^T : \{u_1^T, u_2^T, \ldots, u_k^T = i, \ldots, u_n^T\}.$$

Substituting $p(U_{k0}^T|H_j)$ by $p(U_k^T|H_j) - p(U_{k1}^T|H_j)$, j=0, 1 in (2.47), rearranging and factoring out common terms, we have

$$J(\Gamma) = \sum_{U_k^T} \quad C_f p(U_{k1}^T|H_0)[p(u_0^T = 1|U_{k1}^T) - p(u_0^T = 1|U_{k0}^T)]$$
$$-C_d p(U_{k1}^T|H_1)[p(u_0^T = 1|U_{k1}^T) - p(u_0^T = 1|U_{k0}^T)]$$
$$+p(u_0^T = 1|U_{k0}^T)[C_f p(U_k^T|H_0) - C_d p(U_k^T|H_1)] + C \qquad (2.48)$$

Noting that the last two terms in (2.48) are independent of the optimization of the k^{th} local detector, we drop those terms in the subsequent equations and denote the new cost function by $J^1(\Gamma)$. Letting $p(u_0^T = 1|U_{k1}^T) - p(u_0^T = 1|U_{k0}^T) = f(U_k^T)$ and factoring it out in Equation (2.48), we get

$$J^1(\Gamma) = \sum_{U_K^T} f(U_k^T)[C_f p(U_{k1}^T|H_0) - C_d p(U_{k1}^T|H_1)] \qquad (2.49)$$

Expanding Equation (2.49) in u_0^{T-1} the previous global decision and
$Y^T : (y_1^T, y_2^T, ..., y_n^T)$ the observation vector of local detectors at time step t=T,

$$J^1(\Gamma) = \sum_{U_k^T} f(U_k^T) \sum_{u_0^{T-1}} \quad \int_{Y^T}[C_f p(U_{k1}^T, u_0^{T-1}, Y^T|H_0)$$
$$-C_d p(U_{k1}^T, u_0^{T-1}, Y^T|H_1)] \qquad (2.50)$$

where the integral $\int_{Y^T}$ is multifold integral of dimension n. By conditioning (2.50) on u_0^{T-1} and Y^t, we have

$$J^1(\Gamma) = \sum_{U_k^T} f(U_k^T) \sum_{u_0^{T-1}} \quad \int_{Y^T}[C_f p(U_{k1}^T|u_0^{T-1}, Y^T, H_0)p(Y^T, u_0^{T-1}|H_0)$$
$$-C_d p(U_{k1}^T|u_0^{T-1}, Y^T, H_1)p(Y^T, u_0^{T-1}|H_1)] \qquad (2.51)$$

It is seen that the local decision vector U_{ki}^T given both the previous global decision u_0^{T-1} and the observation vector Y^T does not depend on the hypothesis present. Assuming the observation independence in time, the previous global decision u_0^{T-1} is independent of the observation vector Y^T. In addition, the k^{th} local detector's

decision u_k^T depends on the detector's input and not on other detector decisions. Hence,

$$p(U_{k1}^T|u_0^{T-1}, Y^T) = p(u_k^T = 1|u_0^{T-1}, Y^T) \prod_{i=1, i\neq k}^{n} p(u_i^T|u_0^{T-1}, Y^T).$$

Using the spatial independence of observations the above reduces to

$$p(U_{k1}^T|u_0^{T-1}, Y^T) = p(u_k^T = 1|u_0^{T-1}, y_k^T) \prod_{i=1, i\neq k}^{n} p(u_i^T|u_0^{T-1}, y_i^T).$$

Substituting all of the above results in the cost function $J^1(\Gamma)$ of Equation (2.51), we get

$$\begin{aligned} J^1(\Gamma) = \sum_{U_K^T} \quad & f(U_k^T) \textstyle\sum_{u_0^{T-1}} \int_{Y^T} [C_f p(u_k^T = 1|u_0^{T-1}, y_k^T) \\ & (\textstyle\prod_{i=1, i\neq k}^{n} p(u_i^T|u_0^{T-1}, y_i^T))(\prod_{i=1}^{n} p(y_i^T|H_0)) p(u_0^{T-1}|H_0) \\ & -C_d p(u_k^T = 1|u_0^{T-1}, y_k^T)(\textstyle\prod_{i=1, i\neq k}^{n} p(u_i^T|u_0^{T-1}, y_i^T)) \\ & (\textstyle\prod_{i=1}^{n} p(y_i^T|H_1)) p(u_0^{T-1}|H_1)] \end{aligned} \tag{2.52}$$

Factoring out the common term $p(u_k^T = 1|u_0^{T-1}, y_k^T)$ and rearranging the order of integration and summation, we have

$$\begin{aligned} J^1(\Gamma) = \sum_{u_0^{T-1}} \quad & \textstyle\int_{y_k^T} p(u_k^T = 1|u_0^{T-1}, y_k^T) \sum_{U_k^T} f(U_k^T) \\ & \textstyle\int_{Y_{\bar{k}}^T} [C_f(\prod_{i=1, i\neq k}^{n} p(u_i^T|u_0^{T-1}, y_i^T) p(y_i^T|H_0)) p(y_k^T|H_0) p(u_0^{T-1}|H_0) \\ & -C_d p(y_k^T|H_1) p(u_0^{T-1}|H_1) \textstyle\prod_{i=1, i\neq k}^{n} p(u_i^T|u_0^{T-1}, y_i^T) \\ & p(u_0^{T-1}|H_1)] \end{aligned} \tag{2.53}$$

Integrating over $Y_{\bar{k}}^T$, we rewrite (2.53) using notations of Section 2.2 as

$$\begin{aligned} J^1(\Gamma) = \sum_{u_0^{T-1}} \quad & \textstyle\int_{y_k^T} p(u_k^T = 1|u_0^{T-1}, y_k^T) \sum_{U_{\bar{k}}^T} f(U_k^T) \\ & \times[C_f p(U_{\bar{k}}^T|u_0^{T-1}, H_0) p(y_k^T|H_0) p(u_0^{T-1}|H_0) \\ & -C_d p(U_{\bar{k}}^T|u_0^{T-1}, H_1) p(y_k^T|H_1) p(u_0^{T-1}|H_1)] \end{aligned} \tag{2.54}$$

To minimize the cost function of $J^1(\Gamma)$ in (2.54) we choose

$$p(u_k^T = 1|u_0^{T-1}, y_k^T) = \begin{cases} 1 & if\ A_0 < A_1 \\ 0 & otherwise \end{cases} \tag{2.55}$$

where

$$A_0 = \sum_{U_k^t} f(U_k^T) C_f p(U_k^T|u_0^{T-1}, H_0) p(y_k^T|H_0) p(u_0^{T-1}|H_0)$$
$$A_1 = \sum_{U_k^t} f(U_k^T) C_d p(U_k^T|u_0^{T-1}, H_1) p(y_k^T|H_1) p(u_0^{T-1}|H_1)$$

The k^{th} local decision rule at time step T is therefore given by rewriting (2.55) as:

$$\gamma_k^T(y_k^T, u_0^{T-1}) = \begin{cases} 1 & if\ \dfrac{p(y_k^T|H_1)}{p(y_k^T|H_0)} > \eta_k^T(u_0^{T-1}) \\ 0 & otherwise \end{cases} \tag{2.56}$$

where $\eta_k^T(u_0^{T-1})$ is the k^{th} detector threshold at time step T defined by

$$\eta_k^T(u_0^{T-1}) = \frac{C_f \sum_{U_k^T} f(U_k^T) p(U_k^T|u_0^{T-1}, H_0) p(u_0^{T-1}|H_0)}{C_d \sum_{U_k^T} f(U_k^T) p(U_k^T|u_0^{T-1}, H_1) p(u_0^{T-1}|H_1)} \tag{2.57}$$

as given in Equations (2.44)-(2.46). The local decision rules for time step t<T are derived by recalling Equation (2.41). We write (2.41) explicitly in terms of the k^{th} local decision rule at time step t,

$$\begin{aligned} J^1(\Gamma) = \sum_{U_k^T} \quad & p(u_0^t = 1|U_{k1}^t)\{C_f p(U_{k1}^t|H_0)[p(u_0^T = 1|u_0^t = 1, H_0) \\ & - p(u_0^T = 1|u_0^t = 0, H_0)] - C_d p(U_{k1}^t|H_1)[p(u_0^T = 1|u_0^t = 1, H_1) \\ & - p(u_0^T = 1|u_0^t = 0, H_1)]\} + p(u_0^t = 1|U_{k0}^t)\{C_f p(U_{k0}^t|H_0) \\ & [p(u_0^T = 1|u_0^t = 1, H_0) - p(u_0^T = 1|u_0^t = 0, H_0)] \\ & - C_d p(U_{k0}^t|H_1)[p(u_0^T = 1|u_0^t = 1, H_1) \\ & - p(u_0^T = 1|u_0^t = 0, H_1)]\} \end{aligned} \tag{2.58}$$

Letting $p(u_0^T = 1|u_0^t = 1, H_j) - p(u_0^T = 1|u_0^t = 1, H_j) = g(t,j)$ and substituting $p(U_{k0}^t|H_j)$ *by* $p(U_k^t|H_j) - p(U_{k1}^t|H_j)$, we rewrite (2.58) after rearranging as

$$J^1(\Gamma) = \sum_{U_k^t} \quad p(u_0^t = 1|U_{k1}^t)[C_f p(U_{k1}^t|H_0) g(t,0) - C_d p(U_{k1}^t|H_1) g(t,1)]$$

$$-p(u_0^t = 1|U_{k0}^t)[C_f p(U_{k1}^t|H_0)g(t,0) - C_d p(U_{k1}^t|H_1)g(t,1)]$$

$$+p(u_0^t = 1|U_{k0}^t)[C_f p(U_k^t|H_0)g(t,0) - C_d p(U_k^t|H_1)g(t,1)] \quad (2.59)$$

The last term in the above equation is independent of the optimization of the kth local decision rule, hence we drop that term and denote the new cost function by $J^2(\Gamma)$. Factoring out the resultant common term, Equation (2.59) is written as

$$J^2(\Gamma) = \sum_{U_k^t} [C_f p(U_{k1}^t|H_0)g(t,0) - C_d p(U_{k1}^t|H_1)g(t,1)]$$

$$[p(u_0^t = 1|U_{k1}^t) - p(u_0^t = 1|U_{k0}^t)] \quad (2.60)$$

Next, we expand (2.60) in u_0^{t-1} the previous global decision and Y^t the observation vector of the local detector at time step t to get

$$J^2(\Gamma) = \sum_{U_k^t} \sum_{u_0^{t-1}} \int_{Y^t} [C_f p(U_{k1}^t, u_0^{t-1}, Y^t|H_0)g(t,0)$$

$$-C_d p(U_{k1}^t, u_0^{t-1}, Y^t|H_1)g(t,1)]$$

$$\times[p(u_0^t = 1|U_{k1}^t) - p(u_0^t = 1|U_{k0}^t)] \quad (2.61)$$

Letting $p(u_0^t = 1|U_{k1}^t) - p(u_0^t = 1|U_{k0}^t) = f(U_k^t)$ and conditioning on u_0^{t-1} and Y^t, we get

$$J^2(\Gamma) = \sum_{U_k^t} \sum_{u_0^{t-1}} \int_{Y^t} f(U_k^T)[C_f p(U_{k1}^t|u_0^{t-1}, Y^t, H_0)$$

$$p(u_0^{t-1}, Y^t|H_0)g(t,0) - C_d p(U_{k1}^t|u_0^{t-1}, Y^t, H_1)$$

$$\times p(u_0^{t-1}, Y^t|H_1)g(t,1)] \quad (2.62)$$

The local decision vector U_{k1}^t given both the previous global decision u_0^{t-1} and the observation vector Y^t does not depend on the hypothesis present. Using the observation independence in time, the observation vector Y^t is independent of the previous global decision u_0^{t-1}. Therefore, Equation (2.62) is written as:

$$J^2(\Gamma) = \sum_{U_k^t} \sum_{u_0^{t-1}} f(U_k^t) \int_{Y^t} [C_f p(U_{k1}^t|u_0^{t-1}, Y^t)$$

$$\times p(u_0^{t-1}|H_0)p(Y^t|H_0)g(t,0)$$

$$-C_d p(U_{k1}^t|u_0^{t-1}, Y^t)p(u_0^{t-1}|H_1)p(Y^t|H_1)g(t,1)] \quad (2.63)$$

The k^{th} local detector decision depends on the input of this detector and not on other detector decisions and observations. Moreover, using the spatial independence of observations, we write the term $p(U_{k1}^t|u_0^{t-1}, Y^t)$ as

$$p(U_{k1}^t|u_0^{t-1}, Y^t) = p(u_k^t = 1|u_0^{t-1}, y_k^t) \times \prod_{i=1, i\neq k}^{n} p(u_i^t|u_0^{t-1}, y_i^t).$$

Substituting the above in Equation (2.63) and rearranging the resultant terms, we have

$$\begin{aligned} J^2(\Gamma) = \sum_{U_k^t} \quad & \textstyle\sum_{u_0^{t-1}} f(U_k^t) \int_{Y^t} [C_f p(u_k^t = 1|u_0^{t-1}, y_k^t) p(u_0^{t-1}|H_0) \\ & g(t,0) p(y_k^t|H_0) \textstyle\prod_{i=1, i\neq k}^{n} p(u_i^t|u_0^{t-1}, y_i^t) p(y_i^t|H_0) \\ & - C_d p(u_k^t = 1|u_0^{t-1}, y_k^t) p(u_0^{t-1}|H_1) g(t,1) p(y_k^t|H_1) \\ & \textstyle\prod_{i=1, i\neq k}^{n} p(u_i^t|u_0^{t-1}, y_i^t) p(y_i^t|H_1)] \end{aligned} \tag{2.64}$$

Taking the common factor $p(u_k^t = 1|u_0^{t-1}, y_k^t)$ out and integrating (2.64) over the observation vector Y_k^t only, we get

$$\begin{aligned} J^2(\Gamma) = \sum_{u_0^{t-1}} \quad & \textstyle\int_{y_k^t} p(u_k^t = 1|u_0^{t-1}, y_k^t) \sum_{U_k^t} f(U_k^t) \\ & [C_f p(u_0^{t-1}|H_0) g(t,0) p(y_k^t|H_0) \textstyle\prod_{i=1, i\neq k}^{n} p(u_i^t|u_0^{t-1}, H_0) \\ & - C_d p(u_0^{t-1}|H_1) g(t,1) p(y_k^t|H_1) \textstyle\prod_{i=1, \neq k}^{n} p(u_i^t|u_0^{t-1}, H_1)] \end{aligned} \tag{2.65}$$

To minimize the cost function $J^2(\Gamma)$ of (2.65) we choose

$$p(u_k^t = 1|u_0^{t-1}, y_k^t) = \begin{cases} 1 & if \ A_0 < A_1 \\ 0 & otherwise \end{cases} \tag{2.66}$$

where

$$A_0 = \sum_{U_k^t} f(U_k^t)[C_f p(U_k^t, u_0^{t-1}|H_0) g(t,0) p(y_k^t|H_0)]$$

$$A_1 = \sum_{U_k^t} f(U_k^t)[C_d p(U_k^t, u_0^{t-1}|H_1) g(t,1) p(y_k^t|H_1)]$$

where we substituted $p(U_k^t|u_0^{t-1}, H_j)$ for $\prod_{i=1,i\neq k}^{n} p(u_i^t|u_0^{t-1}, H_j)$ and combined $p(U_k^t|u_0^{t-1}, H_j)$ and $p(u_0^{t-1}|H_j)$ to yield $p(U_k^t, u_0^{t-1}|H_j)$. Therefore, the k^{th} detector decision rule at time t<T is given by rewriting Equation (2.66) as:

$$\gamma_k^t(y_k^t, u_0^{t-1}) = u_k^t = \begin{cases} 1 & if \ \dfrac{p(y_k^t|H_1)}{p(y_k^t|H_0)} > \eta_k^t(u_0^{t-1}) \\ 0 & otherwise \end{cases} \tag{2.67}$$

where $\eta_k^t(u_0^{t-1})$ is the k^{th} detector threshold at time step t defined as:

$$\eta_k^t(u_0^{t-1}) = \frac{C_f g(t,0) \sum_{U_k^t} f(U_k^t) p(U_k^t, u_0^{t-1}|H_0)}{C_d g(t,1) \sum_{U_k^t} f(U_k^t) p(U_k^t, u_0^{t-1}|H_1)}$$

as stated in Theorem 2.4.

Q.E.D.

It should be noticed that there are two different threshold equations for the local likelihood ratio test. The first equation is (2.45) for time step t=T and the second equation is (2.46) for time step t<T. Similarly, the global decision rule has a threshold of C_f/C_d at time t=T and $C_f g(t,0)/C_d g(t,1)$ at time t<T as stated in Theorem 2.3. The system thresholds up to time T are found by simultaneously solving the set of threshold equations given by (2.45) and (2.46) for all time t, t$\leq$ T.

System Performance

Next, we present the performance equations for the system namely the system probability of error $p_{e_0}^t$. As before, the system probability of error at time T is given by

$$p_{e_0}^T = p_{f_0}^T p(H_0) + p_{m_0}^T p(H_1). \tag{2.68}$$

The system probability of false alarm and miss, i.e., $p_{f_0}^t, p_{m_0}^t$, are given by Equations (2.22) and (2.25) respectively. We recall those equations

$$p_{f_0}^t = p_{f_0}^{t-1}[p_{f_0}^t(u_0^{t-1}=1) - p_{f_0}^t(u_0^{t-1}=0)] + p_{f_0}^t(u_0^{t-1}=0) \tag{2.69}$$

$$p_{m_0}^t = p_{m_0}^{t-1}[p_{m_0}^t(u_0^{t-1} = 0) - p_{m_0}^t(u_0^{t-1} = 1)] + p_{m_0}^t(u_0^{t-1} = 1) \tag{2.70}$$

where

$$p_{m_0}^t(u_0^{t-1} = i) = \sum_{U^t} p(u_0^t = 0|U^t)p(U^t|u_0^{t-1} = i, H_1) \tag{2.71}$$

$$P_{f_0}^t(u_0^{t-1} = i) = \sum_{U^t} p(u_0^t = 1|U^t)p(U^t|u_0^{t-1} = i, H_0). \tag{2.72}$$

The performance of the system is found when t=T. Hence, the system probability of miss and false alarm have to be computed recursively up to time step t=T. Time step t=1 represents a special case where the global decision rule is the same as given in Equation (2.32) of Theorem 2.3 with t=1. The local decision rule is the same as in Equation (2.44) of Theorem 2.4 with the local threshold equation of (2.46) modified for t=1 as follows

$$\eta_k^1 = \frac{C_f g(t=1,0) \sum_{U_k^1} f(U_k^1)p(U_k^1|H_0)}{C_d g(t=1,1) \sum_{U_k^1} f(U_k^1)p(U_k^1|H_1)} \tag{2.73}$$

which is obtained by dropping the previous global decision term u_0^{t-1} in Equation (2.46).

It is seen that threshold equation of the k^{th} detector is coupled with other detector thresholds at time step t, i.e., we have spatial coupling. In addition, there is a temporal coupling of threshold equations through the term g(t=i, 0). Hence, we have a set of non linear threshold equations that are coupled spatially and temporally. For a given time t=T, the computational complexity appears to inhibit a numerical solution; hence, in the next section we consider a simpler system consisting of only one detector with feedback, thereby eliminating the spatial coupling with other detectors.

2.5 The Single Detector with Feedback

We consider the single detector system with feedback shown in Figure 2.8. In this system, we only have a single detector and do not have separate global and local detectors. Therefore, the results obtained in the previous section cannot be used directly. Using the notations defined earlier and dropping the subscripts since there is one detector only, we derive the FSS decision rules for this system next.

THEOREM 2.5

For the one detector with feedback shown in Figure 2.8, the decision rules that minimize the Bayesian cost function in the binary hypothesis testing problem with a fixed sample size is given by

$$\gamma^1(y^1) = u^1 = \begin{cases} 1 & if\ \Lambda(y^1) > C_f g(1,0)/C_d g(1,1) \\ 0 & otherwise \end{cases} \tag{2.74}$$

and for t > 1

$$\gamma^t(y^t, u^{t-1}) = u^t = \begin{cases} 1 & if\ \Lambda(y^t) > \dfrac{C_f g(t,0) p(u^{t-1}|H_0)}{C_d g(t,1) p(u^{t-1}|H_1)} \\ 0 & otherwise \end{cases} \tag{2.75}$$

where

$$g(t,j) = p(u^T = 1|u^t = 1, H_j) - p(u^T = 1|u^t = 0, H_j).$$

Proof:

Recall the Bayesian cost function to be minimized

$$J(\Gamma) = C_f p_{f_0}^T - C_d p_{d_0}^T + C \tag{2.76}$$

where C_f, C_d and C are as defined before. We expand $p_{f_0}^T$ and $p_{d_0}^T$ in terms of u^t

$$J(\Gamma) = C_f \sum_{u^t} p(u^T = 1, u^t|H_0) - C_d \sum_{u^t} p(u^T = 1, u^t|H_1) + C. \tag{2.77}$$

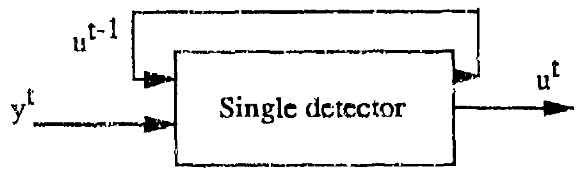

Fig. 2.8: The single detector system with feedback.

Conditioning on u^t and expanding (2.77), we get

$$J(\Gamma) = \sum_{u^t} C_f p(u^T = 1|u^t, H_0)p(u^t|H_0) - C_d p(u^T = 1|u^t, H_1)p(u^t|H_1) + C \quad (2.78)$$

Writing the above equation explicitly in term of all possibilities of u^t, namely u^t=0, 1 and substituting $p(u^t = 0|H_j) = 1 - p(u^t = 1|H_j)$, we get,

$$\begin{aligned} J(\Gamma) = \quad & C_f p(u^T = 1|u^t = 1, H_0)p(u^t = 1|H_0) \\ & + C_f p(u^T = 1|u^t = 0, H_0)(1 - p(u^t = 1|H_0)) \\ & - C_d p(u^T = 1|u^t = 1, H_1)p(u^t = 1|H_1) \\ & - C_d p(u^T = 1|u^t = 0, H_1)(1 - p(u^t = 1|H_1)) + C \end{aligned} \quad (2.79)$$

Multiplying out, factoring the common term $p(u^t = 1|H_j)$ out and rearranging, we get

$$\begin{aligned} J(\Gamma) = \quad & C_f p(u^t = 1|H_0)[p(u^T = 1|u^t = 1, H_0) - p(u^T = 1|u^t = 0, H_0)] \\ & - C_d p(u^t = 1|H_1)[p(u^T = 1|u^t = 1, H_1) - p(u^T = 1|u^t = 0, H_1)] \\ & + C_f p(u^T = 1|u^t = 0, H_0) - C_d p(u^T = 1|u^t = 0, H_1) + C \end{aligned} \quad (2.80)$$

The last three terms are independent of the optimization of the decision rule at time t. Hence, we drop these terms and denote the new cost function by $J^1(\Gamma)$. Letting $p(u^T = 1|u^t = 1, H_j) - p(u^T = 1|u^t = 0, H_j) = g(t, j)$, we rewrite Equation (2.80) as

$$J^1(\Gamma) = C_f p(u^t = 1|H_0)g(t, 0) - C_d p(u^t = 1|H_1)g(t, 1) \quad (2.81)$$

Introducing the observations y^t and using the law of total probability, we have

$$J^1(\Gamma) = \int_{y^t} C_f g(t, 0)p(u^t = 1, y^t|H_0) - C_d g(t, 1)p(u^t = 1, y^t|H_1) \quad (2.82)$$

Conditioning on y^t and expanding the above we get

$$\begin{aligned} J^1(\Gamma) = \quad & \int_{y^t} C_f g(t, 0)p(u^t = 1|y^t, H_0)p(y^t|H_0) \\ & - C_d g(t, 1)p(u^t = 1|y^t, H_1)p(y^t|H_1) \end{aligned} \quad (2.83)$$

Letting t=1 in the above equation and observing that the detector decision u^t given the observation y^t does not depend on the hypothesis present, we factor out the resulting common term and rewrite Equation (2.83) as

$$J^1(\Gamma) = \int_{y^t} p(u^1 = 1|y^1)[C_f g(t = 1,0)p(y^1|H_0) - C_d g(t = 1,1)p(y^1|H_1)] \qquad (2.84)$$

To minimize the cost function $J^1(\Gamma)$ of Equation (2.84) we choose

$$p(u^1 = 1|y^1) = \begin{cases} 1 \; if & C_f g(t = 1,0)p(y^1|H_0) < \\ & C_d g(t = 1,1)p(y^1|H_1) \\ 0 & otherwise. \end{cases} \qquad (2.85)$$

which is the decision rule at time t=1 as given in Equation (2.74) of Theorem 2.5. We proceed to derive the rest of decision rules for t>1 by expanding Equation (2.82) in terms of the previous detector decision u^{t-1} as follows:

$$\begin{aligned} J^1(\Gamma) = \int_{y^t} \quad & \textstyle\sum_{u^{t-1}} C_f p(u^t = 1, u^{t-1}, y^t|H_0)g(t,0) \\ & - C_d p(u^t = 1, u^{t-1}, y^t|H_1)g(t,1) \end{aligned} \qquad (2.86)$$

Conditioning on u^{t-1} and y^t and expanding, we have

$$\begin{aligned} J^1(\Gamma) = \int_{y^t} \sum_{u^{t-1}} \quad & C_f p(u^t = 1|u^{t-1}, y^t, H_0)p(u^{t-1}, y^t|H_0)g(t,0) \\ & -C_d p(u^t = 1|u^{t-1}, y^t, H_1)p(u^{t-1}, y^t|H_1)g(t,1) \end{aligned} \qquad (2.87)$$

The decision u^t given the observation y^t and the previous decision u^{t-1} does not depend on the hypothesis present. Therefore, we rewrite (2.87), after factoring the common term $p(u^t = 1|u^{t-1}, y^t)$ out, as

$$\begin{aligned} J^1(\Gamma) = \int_{y^t} \sum_{u^{t-1}} \quad & p(u^t = 1|u^{t-1}, y^t)[C_f g(t,0)p(y^t, u^{t-1}|H_0) \\ & -C_d g(t,1)p(y^t, u^{t-1}|H_1)] \end{aligned} \qquad (2.88)$$

To minimize the cost function $J^1(\Gamma)$ of (2.88) we choose

$$p(u^t = 1|u^{t-1}, y^t) = \begin{cases} 1 & if \; C_f g(t,0)p(y^t, u^{t-1}|H_0) < \\ & C_d g(t,1)p(y^t, u^{t-1}|H_1) \\ 0 & otherwise. \end{cases} \qquad (2.89)$$

as stated in Equation (2.89).

Q.E.D.

At time t=T, we have g(t=T, j) = 1. This results in the decision rule at time t=T defined as:

$$u^T = \gamma^T(y^T, u^{T-1}) = \begin{cases} 1 & if\ \Lambda(y^T) > \dfrac{C_f p(u^{T-1}|H_0)}{C_d p(u^{T-1}|H_1)} \\ 0 & otherwise \end{cases} \tag{2.90}$$

The probability of error for this system is given by Equation (2.68) with the probability of miss and false alarm as given by (2.69) and (2.70). However

$$p_m^t(u^{t-1} = i) = \int_{\lambda^t(u^{t-1}=i)}^{\infty} dF_1 \tag{2.91}$$

$$p_f^t(u^{t-1} = i) = \int_{\lambda^t(u^{t-1}=i)}^{\infty} dF_0 \tag{2.92}$$

where F_1 and F_0 are the conditional probability distributions under the hypotheses H_1 and H_0 respectively. The quantity $\lambda^t(u^{t-1} = i)$ is the threshold to be used when integrating over the probability densities. It is related to the threshold $\eta^t(u^{t-1} = i)$ in an obvious manner.

In the next section, we apply the results obtained in this section to serial networks.

2.6 Detection Results for the Serial Network

In this section, we present the design and analysis of another important class of decentralized detection networks namely the serial (tandem) network. This class of networks has been investigated in the literature [8, 17, 11]. We show the similarities of the serial network with the decentralized detection system with feedback studied previously.

Consider a serial system consisting of N detectors shown in Figure 2.9. Based on its observation, the first detector makes a decision regarding the hypothesis present and transmits it to the second detector. The second detector bases its decision on the decision of the first detector and its own observation. This decision is transmitted to the third detector. This process continues until the final detector which yields the global decision. This serial system can be viewed as a single detector system with feedback discussed in Section 2.5 with t=n. Thus, the threshold equations for the serial system can be written by substituting t=n and T=N in Equations (2.74) and (2.75) of Theorem 2.5. The results are presented in Lemma 2.1 next.

Lemma 2.1

For a serial system consisting of N detectors as shown in Figure 2.9, the n^{th} detector decision rule that minimizes the Bayesian cost function in the binary hypothesis testing problem is given by:

$$\gamma^1(y^1) = u^1 = \begin{cases} 1 & if\ \Lambda(y^1) > \dfrac{C_f g(1,0)}{C_d g(1,1)} \\ 0 & otherwise \end{cases} \tag{2.93}$$

and for n > 1

$$\gamma^n(y^n, u^{n-1}) = u^n = \begin{cases} 1 & if\ \Lambda(y^n) > \dfrac{C_f g(n,0) p(u^{n-1}|H_0)}{C_d g(n,1) p(u^{n-1}|H_1)} \\ 0 & otherwise \end{cases} \tag{2.94}$$

where

$$g(n,j) = p(u^N = 1|u^n = 1, H_j) - p(u^N = 1|u^n = 0, H_j)$$

Proof:

A direct substitution of t=1 and T=N in Equations (2.93) and (2.94) results in Equations (2.74) and (2.75).

Q.E.D.

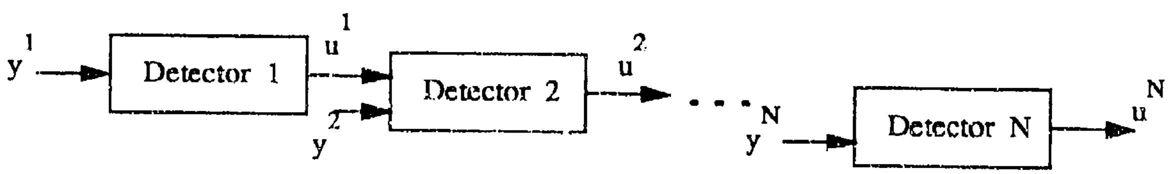

Fig. 2.9: A serial system consisting of N detectors.

In order to demonstrate the validity of our results, we consider the case of 3 detectors in tandem and show that our results agree with the results established in the literature [8]. For N=3, the decision rule of the first detector is given by:

$$\gamma^1(y^1) = u^1 = \begin{cases} 1 & if\ \Lambda(y^1) > \eta^1 \\ 0 & otherwise \end{cases} \tag{2.95}$$

where η^1 is the threshold of the first detector defined as:

$$\eta^1 = \frac{C_f[p(u^3 = 1|u^1 = 1, H_0) - p(u^3 = 1|u^1 = 0, H_0)]}{C_d[p(u^3 = 1|u^1 = 1, H_1) - p(u^3 = 1|u^1 = 0, H_1)]}. \tag{2.96}$$

The decision rule of the second detector is given by:

$$\gamma^2(y^2, u^1) = u^2 = \begin{cases} 1 & if\ \Lambda(y^2) > \eta^2(u^1) \\ 0 & otherwise \end{cases} \tag{2.97}$$

where $\eta^2(u^1)$ is the threshold of the second detector defined as:

$$\eta^2(u^1) = [\frac{p(u^3 = 1|u^2 = 1, H_0) - p(u^3 = 1|u^2 = 0, H_0)}{p(u^3 = 1|u^2 = 1, H_1) - p(u^3 = 1|u^2 = 0, H_1)}]\frac{C_f p(u^1|H_0)}{C_d p(u^1|H_1)}.$$

The decision rule of the third detector is:

$$\gamma^3(y^3, u^2) = u^3 = \begin{cases} 1 & if\ \Lambda(y^3) > \eta^3(u^2) \\ 0 & otherwise \end{cases} \tag{2.98}$$

where $\eta^3(u^2)$ is the threshold of the third detector defined as:

$$\eta^3(u^2) = \frac{C_f p(u^2|H_0)}{C_d p(u^2|H_1)}$$

It is seen that these decision rules are the same as those of Reibman and Nolte [8]. Analogously, solving the single detector with feedback problem up to time t=T corresponds to solving the problem of N detectors in tandem. Moreover, the decison rule at time step $t \leq T$ in the single detector with feedback corresponds to the decision rule of the n^{th} detector ($n \leq N$) in the tandem network.

Having established the correspondence between the serial network and the single detector with feedback, the rest of our work on decentralized detection systems with feedback could be applied to more complicated configurations such as the one shown in Figure 2.10. In this system configuration, the block of n detectors and a fusion center is repeated T times with the decision of each block feeding into the next block. The decision rules for this tandem configuration are given by the decision rules for the decentralized detection system with feedback given in Theorems 2.3 and 2.4 with time step t corresponding to the t^{th} block in the tandem network. Hence, the tandem configuration of Figure 2.10 is equivalent to the decentralized detection system with feedback with the t^{th} block thresholds of the tandem network being the same as the t^{th} time step thresholds of the decentralized detection system with feedback. If the decision rules obtained in Section 2.3 are used for the system shown in Figure 2.10, then the interpretation is that each detector block of the tandem network attempts to optimize itself rather than trying to optimize the entire system.

2.7 Discussion

In this chapter, we presented the Bayesian formulation of a decentralized detection system with feedback. Two cases were considered namely the FSS problem and the less restrictive problem of the system without any a priori knowledge of the stopping time. Local detector thresholds were shown to be a function of the previous global decision. It was shown that a serial detection system can be interpreted as a single detector system with feedback. Numerical results for system performance for the case of unknown stopping time were obtained. Numerical results showed that a performance advantage of the decentralized detection system with feedback considered in this chapter over the corresponding decentralized detection system without feedback cannot be established, in general. In the next chapter, therefore,

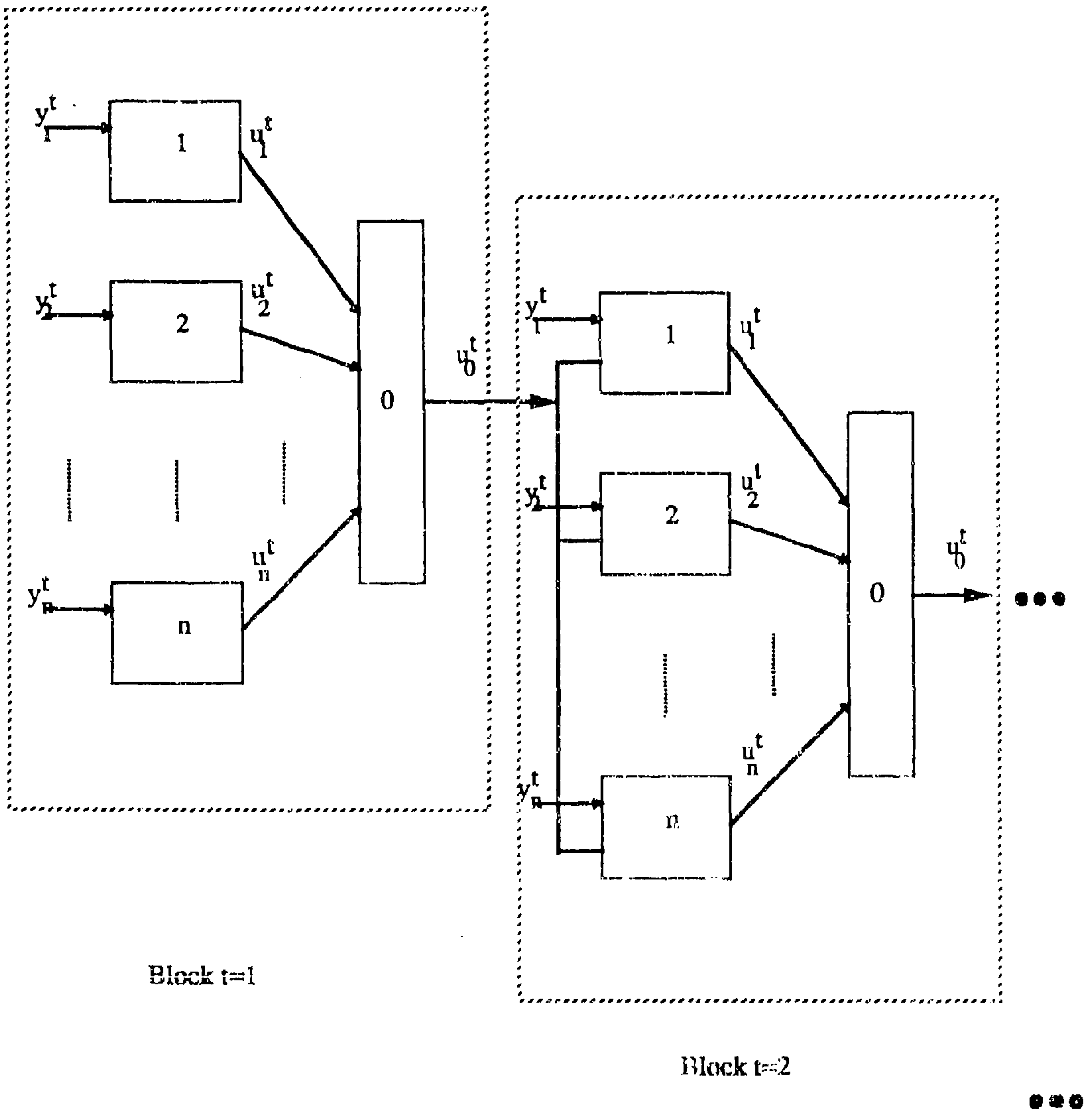

Fig. 2.10: Tandem block of detectors.

we enhance the system and incorporate memory into the decentralized detection system with feedback. We prove analytically that the system with memory outperformes the decentralized detection system without feedback investigated in the literature.

Chapter 3

Decentralized Detection Systems with Feedback and Memory

3.1 Introduction

In the previous chapter, we have considered the decentralized detection system with feedback shown in Figure 2.1. In that system, at any time t each local detector operated only on its current observation y_k^t and the previous global decision u_0^{t-1}. In other words, at time step t, all previous observations $y_k^1, y_k^2, \ldots, y_k^{t-1}$ were discarded. In this chapter, we generalize the system of Figure 2.1 to include the previous observations in the processing at the local detectors as shown in Figure 3.1, i.e., we incorporate memory at the local detectors in the decentralized detection system with feedback. We show that this system with memory and feedback outperforms the conventional decentralized detection system without feedback shown in Figure 3.2.

In Section 3.2, we consider the generalization of the decentralized detection sys-

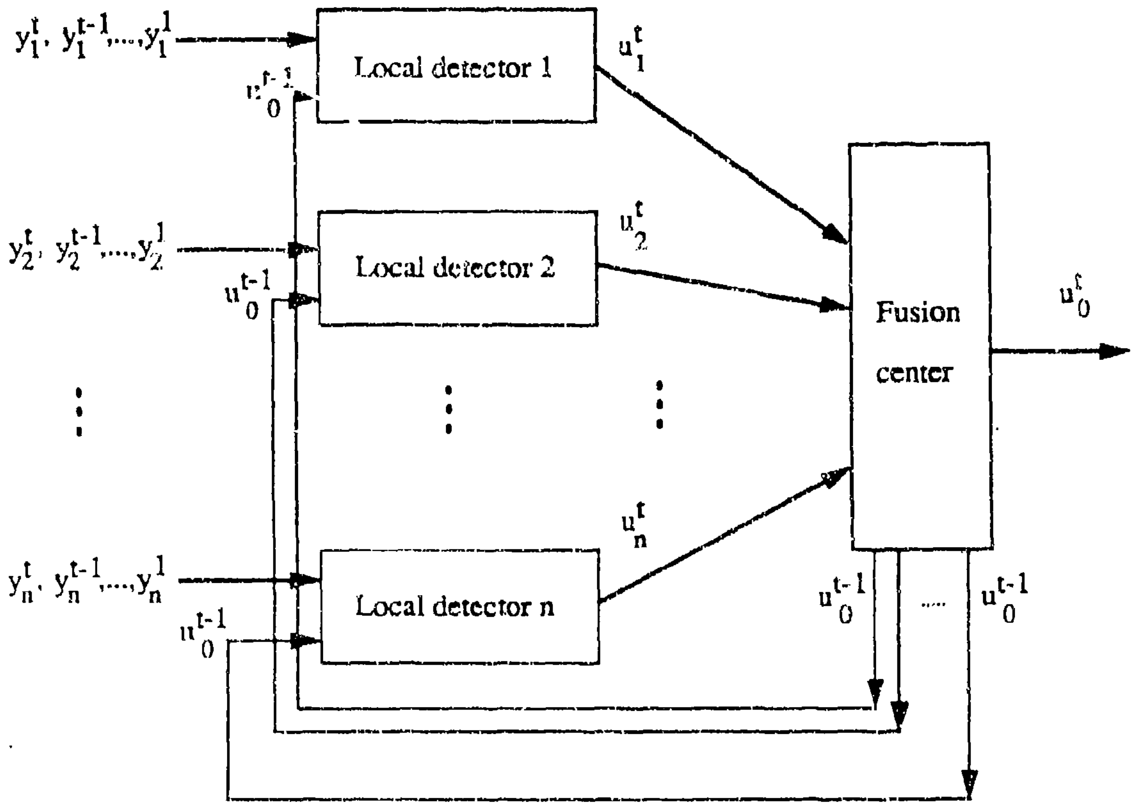

Fig. 3.1: A decentralized detection system with feedback and memory.

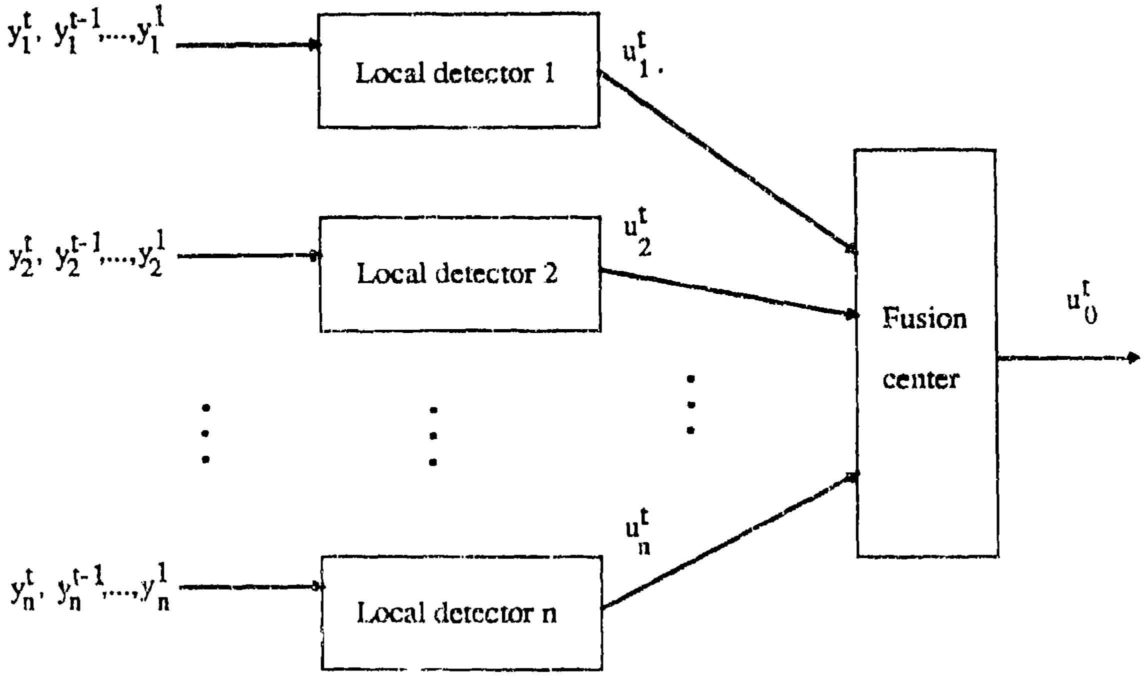

Fig. 3.2: A decentralized detection system with t samples per detector.

tem with feedback of Chapter 2 by incorporating memory at the local detectors. The local detector operates on the previous global decision, its current observation y_k^t and all previous observations $y_k^1, y_k^2, \ldots, y_k^{t-1}$ to produce the local decision u_k^t as shown in Figure 3.1. We formulate the Bayesian hypothesis testing problem for this system. Using the PBPO solution methodology, we derive the optimal fusion rule and local decision rules. The system performance is evaluated and compared to the performance of a decentralized detection system without feedback shown in Figure 3.2. The asymptotic performance of the decentralized detection system with feedback is investigated and the probability of system error is shown to go to zero asymptotically. In Section 3.3, we study the data transmission requirements for the system where due to the feedback links additional data transmission is required. Techniques are developed such that the data transmission requirements are reduced. We propose and study two protocols. Numerical examples are presented in each section.

3.2 System Description and Problem Statement

In this section, we consider the binary hypothesis testing problem for the system shown in Figure 3.1. This system consists of n local detectors which communicate their decisions to the fusion center. The fusion center communicates the global decision back to each of the n detectors. The system operation is described as follows: At time step t, the k^{th} detector makes the local decision u_k^t, k=1, 2, ..., n, based on the previous global decision u_0^{t-1}, the previous observations $y_k^{t-1}, y_k^{t-2}, \ldots, y_k^1$ denoted by $Y_{t-1,k}$ and the current observation y_k^t. The local decision u_k^t is transmitted to the fusion center where it is combined with the other incoming local decisions to yield the global decision u_0^t. The global decision u_0^t is fed back to all the local detectors for use at the next time step t+1. We assume that the joint conditional probability density functions $p(Y^t, Y^{t-1}, ..., Y^1|H_j)$, j=0, 1 are known

a priori where Y^t is the concatenation of all local observations at time step t, i.e. $Y^t = \{y_1^t, y_2^t, \ldots, y_n^t\}$. The local decision u_k^t is obtained using the decision rule $\gamma_k^t(.)$ as follows

$$u_k^t = \gamma_k^t(Y_{t,k}, u_0^{t-1})$$

where $Y_{t,k} = \{y_k^t, y_k^{t-1}, \ldots, y_k^1\}$.

The global decision u_0^t is obtained using the global decision rule $\gamma_0^t(.)$ as follows:

$$u_0^t = \gamma_0^t(U^t)$$

where $U^t = \{u_1^t, u_2^t, \ldots, u_n^t\}$.

The problem is to find the PBPO decision rules $\gamma_k^t(.)$ for each detector k=0, 1, ..., n, so as to minimize a given cost function $J(\Gamma)$. For the Bayesian formulation, the cost function $J(\Gamma)$ is given by,

$$\begin{aligned} J(\Gamma) = \quad & C_{00}p(u_0^t = 0, H_0) + C_{01}p(u_0^t = 0, H_1) \\ & + C_{10}p(u_0^t = 1, H_0) + C_{11}p(u_0^t = 1, H_1) \end{aligned} \tag{3.1}$$

where C_{ij}, i, j=0, 1, is the cost of deciding $u_0^t = H_i$ when the true hypothesis is H_j. The costs C_{ij}, i, j=0, 1 and the a priori probabilities $p(H_0)$ and $p(H_1)$ are assumed to be known. Rewriting (3.1) in terms of the probability of false alarm at time step t, $p_{f_0}^t$, and the probability of detection at time step t, $p_{d_0}^t$, we have

$$J(\Gamma) = C_f p_{f_0}^t - C_d p_{d_0}^t + C \tag{3.2}$$

where

$C_f = P(H_0)(C_{10} - C_{00})$

$C_d = P(H_1)(C_{01} - C_{11})$

$C = P(H_0)C_{00} + P(H_1)C_{01}$

In the next section, we derive the decision rules $\gamma_k^t(.)$ for k=0, 1, ..., n, and evaluate the system performance.

3.3 System Optimization and Performance

Before we proceed with the system optimization, we make certain simplifying assumptions. We assume spatial independence, i.e., the observations at the k^{th} detector denoted by $Y_{t,k} = \{y_k^t, y_k^{t-1}, ..., y_k^1\}$ are statistically independent of the observations at the j^{th} detector (j≠k). Therefore, the a priori knowledge of the conditional probability density functions $p(Y^t, Y^{t-1}, ..., Y^1|H_j)$, j=0, 1, reduces to the a priori knowledge of the individual detector conditional probability densities $p(y_k^t, y_k^{t-1}, ..., y_k^1|H_j)$, k=1, 2, ..., n; j=0, 1. In addition, we assume that the observations at the k^{th} detector, $y_k^t, y_k^{t-1}, ..., y_k^1$ are independent in time. Thus, the a priori knowledge of the individual detector conditional probability density reduces further to the knowledge of the conditional probability densities $p(y_k^t|H_j)$, k=1, 2, ..., n; j=0,1; t=1, 2,... .

Next, we proceed with the minimization of the cost function given in Equation (3.2). Using the PBPO design methodology, the optimal fusion rule $\gamma_0^t(.)$ that minimizes the cost function is derived. The result is presented in Theorem 3.1. Assuming the knowledge of the fusion rule, the local decision rules $\gamma_k^t(.)$, k=1, 2, ..., n, that minimize the cost function of Equation (3.2) are derived in Theorem 3.2.

THEOREM 3.1

For the decentralized detection system with feedback of Figure 3.1, the PBPO fusion rule for the Bayesian binary hypothesis testing problem is given by

$$\gamma_0^t(U^t) = u_0^t = \begin{cases} 1 & if\ \Lambda(U^t) > \frac{C_f}{C_d} \\ 0 & otherwise \end{cases} \tag{3.3}$$

where

$$\Lambda(U^t) = \frac{p(U^t|H_1)}{p(U^t|H_0)} : \textit{the likelihood ratio.}$$

Proof:

Since the fusion center operation is identical to that of the system considered in Chapter 2, the result and the proof are identical to those in Theorem 2.1 of Chapter 2.

Q.E.D.

THEOREM 3.2

The PBPO decision rule at the k^{th} detector for the Bayesian binary hypothesis testing problem is given by

$$\gamma_k^t(Y_{t,k}, u_0^{t-1}) = u_k^t = \begin{cases} 1 & if\ \Lambda(Y_{t,k}) > \eta_k^t(u_0^{t-1}) \\ 0 & otherwise \end{cases} \tag{3.4}$$

where $\eta_k^t(u_0^{t-1})$ is the threshold of the k^{th} detector at time step t defined as:

$$\eta_k^t(u_0^{t-1}) = \frac{C_f \sum_{U_k^t} f(U_k^t) p(U_k^t, u_0^{t-1}|H_0)}{C_d \sum_{U_k^t} f(U_k^t) p(U_k^t, u_0^{t-1}|H_1)} \tag{3.5}$$

and

$$f(U_k^t) = p(u_0^t = 1|U_{k1}^t) - p(u_0^t = 1|U_{k0}^t).$$

Proof:

Recall Equation (2.4) of Theorem 2.1

$$J(\Gamma^t) = \sum_{U^t} p(u_0^t = 1|U^t)[C_f p(U^t|H_0) - C_d p(U^t|H_1)] + C. \tag{3.6}$$

We write (3.6) explicitly in terms of the k^{th} local decision

$$J(\Gamma^t) = \sum_{U_k^t} \quad p(u_0^t = 1|U_{k1}^t)[C_f p(U_{k1}^t|H_0) - C_d p(U_{k1}^t|H_0)]$$
$$+ p(u_0^t = 1|U_{k0})[C_f p(U_{k0}^t|H_0) - C_d p(U_{k0}^t|H_0)] + C \tag{3.7}$$

where

$$U_k^t = \{u_1^t, u_2^t, ..., u_{k-1}^t, u_{k+1}^t, ..., u_n^t\}.$$

$$U_{ki}^t = \{u_1^t, u_2^t, ..., u_k^t = i, ..., u_n^t\}.$$

Substituting $p(U_{k0}^t|H_j) = p(U_k^t|H_j) - p(U_{k1}^t|H_j)$, j=0, 1, in (3.7) and factoring out common terms, we have

$$\begin{aligned} J(\Gamma^t) = \quad & \textstyle\sum_{U_k^t} p(u_0^t = 1|U_{k1}^t)[C_f p(U_{k1}^t|H_0) - C_d p(U_{k1}^t|H_1)] \\ & -p(u_0^t = 1|U_{k0}^t)[C_f p(U_{k1}^t|H_0) - C_d p(U_{k1}^t|H_1)] \\ & +p(u_0^t = 1|U_{k0}^t)[C_f p(U_k^t|H_0) - C_d p(U_k^t|H_1)] + C \end{aligned}$$

Rearranging terms,

$$\begin{aligned} J(\Gamma^t) = \sum_{U_k^t} \quad & [p(u_0^t = 1|U_{k1}^t) - p(u_0^t = 1|U_{k0}^t)] \\ & \times[C_f p(U_{k1}^t|H_0) - C_d p(U_{k1}^t|H_1)] \\ & +p(u_0^t = 1|U_{k0}^t)[C_f p(U_k^t|H_0) - C_d p(U_k^t|H_1)] + C \end{aligned} \tag{3.8}$$

It should be noticed that the proof up to this stage is the same as the proof of Theorem 2.1 since the development is independent of the observation variable. Proceeding with the proof, we observe that the last two terms of (3.8) are not involved in the optimization of the k^{th} local detector. We discard these terms in the subsequent equations and denote the now cost function by $J^1(\Gamma^t)$. Next, we expand (3.8) in u_0^{t-1} the previous global decision, and $Y_t = \{Y^t, Y^{t-1}, ..., Y^1\}$ the observation vectors of local detectors up to time step t, hence

$$\begin{aligned} J^1(\Gamma^t) = \sum_{U_k^t} \quad & [p(u_0^t = 1|U_{k1}^t) - p(u_0^t = 1|U_{k0}^t)] \\ & \textstyle\int_{Y_t} \sum_{u_0^{t-1}} [C_f p(U_{k1}^t, u_0^{t-1}, Y_t|H_0) - C_d p(U_{k1}^t, u_0^{t-1}, Y_t|H_1)] \end{aligned} \tag{3.9}$$

where $\int_{Y_t}$ is a multifold integral over all y_k^t for all k and all time steps up to and including t.

Letting $p(u_0^t = 1|U_{k1}^t) - p(u_0^t = 1|U_{k0}^t) = f(U_k^t)$ and expanding (3.9) by conditioning

on u_0^{t-1} and Y_t, we have

$$J^1(\Gamma^t) = \sum_{U_k^t} f(U_k^t) \int_{Y_t} \sum_{u_0^{t-1}} [C_f p(U_{k1}^t | u_0^{t-1}, Y_t, H_0)$$
$$p(u_0^{t-1}, Y_t | H_0) - C_d p(U_{k1}^t | u_0^{t-1}, Y_t, H_1) p(u_0^{t-1}, Y_t | H_1)] \quad (3.10)$$

The local decision vector U_{k1}^t given both the previous global decision u_0^{t-1} and the observation vector Y_t does not depend on the hypothesis present. In addition, expanding $p(u_0^{t-1}, Y_t | H_j)$ by conditioning on Y_t, we have

$$J^1(\Gamma^t) = \sum_{U_k^t} f(U_k^t) \int_{Y_t} \sum_{u_0^{t-1}}$$
$$[C_f p(U_{k1}^t | u_0^{t-1}, Y_t) p(u_0^{t-1} | Y_t, H_0) p(Y^t, Y_{t-1} | H_0)$$
$$-C_d p(U_{k1}^t | u_0^{t-1}, Y_t) p(u_0^{t-1} | Y_t, H_1) p(Y^t, Y_{t-1} | H_1)] \quad (3.11)$$

where we have used the fact that $Y_t = \{Y^t, Y_{t-1}\}$.

Due to the temporal independence of the observations, the previous global decision u_0^{t-1} does not depend on the observation vector Y^t. Furthermore, using the spatial independence of observations, we rewrite Equation (3.11) in terms of the individual detector's observation vector $Y_{t,k}$,

$$J^1(\Gamma^t) = \sum_{U_k^t} f(U_k^t) \int_{Y_{t,1}} \cdots \int_{Y_{t,n}} \sum_{u_0^{t-1}}$$
$$[C_f p(U_{k1}^t | u_0^{t-1}, Y_t) p(u_0^{t-1} | Y_{t-1}, H_0) \textstyle\prod_{i=1}^n p(Y_{t,i} | H_0)$$
$$-C_d p(U_{k1}^t | u_0^{t-1}, Y_t) p(u_0^{t-1} | Y_{t-1}, H_1) \textstyle\prod_{i=1}^n p(Y_{t,i} | H_1)] \quad (3.12)$$

Since the decision of the k^{th} detector u_k^t depends only on its input observation and does not depend on other detector decisions,

$$p(U_{k1}^t | u_0^{t-1}, Y_t) = p(u_k^t = 1 | u_0^{t-1}, Y_t) \prod_{i=1, i \neq k}^{n} p(u_i^t | u_0^{t-1}, Y_t)$$

Furthermore, due to spatial independence of observations, the above is written as:

$$p(U_{k1}^t | u_0^{t-1}, Y_t) = p(u_k^t = 1 | u_0^{t-1}, Y_{t,k}) \prod_{i=1, i \neq k}^{n} p(u_i^t | u_0^{t-1}, Y_{t,i})$$

Substituting this result in (3.12), factoring out the k^{th} local decision term and rearranging, we have

$$J^1(\Gamma^t) = \sum_{U_k^t} \sum_{u_0^{t-1}} f(U_k^t) \int_{Y_{t,k}} p(u_k^t = 1|u_0^{t-1}, Y_{t,k})$$
$$\int_{Y_{t,1}} \cdots \int_{Y_{t,k-1}} \int_{Y_{t,k+1}} \cdots \int_{Y_{t,n}} [C_f p(u_0^{t-1}|Y_{t-1}, H_0) p(Y_{t,k}|H_0)$$
$$\times [\prod_{i=1,i\neq k}^{n} p(Y_{t,i}|H_0)][\prod_{i=1,i\neq k}^{n} p(u_i^t|u_0^{t-1}, Y_{t,i})]$$
$$-C_d p(u_0^{t-1}|Y_{t-1}, H_1) p(Y_{t,k}|H_1)[\prod_{i=1,i\neq k}^{n} p(Y_{t,i}|H_1)]$$
$$\prod_{i=1,i\neq k}^{n} p(u_i^t|u_0^{t-1}, Y_{t,i})] \tag{3.13}$$

Combining the multiplicative terms in (3.13) and unconditioning on $Y_{t,i}$, we have

$$J^1(\Gamma^t) = \sum_{U_k^t} \sum_{u_0^{t-1}} f(U_k^t) \int_{Y_{t,k}} p(u_k^t = 1|u_0^{t-1}, Y_{t,k})$$
$$\int_{Y_{t,1}} \cdots \int_{Y_{t,k-1}} \int_{Y_{t,k+1}} \cdots \int_{Y_{t,n}} [C_f p(u_0^{t-1}|H_0) p(Y_{t,k}|H_0)$$
$$\times [\prod_{i=1,i\neq k}^{n} p(Y_{t,i}, u_i^t|u_0^{t-1}, H_0)] - C_d p(u_0^{t-1}|H_1)$$
$$p(Y_{t,k}|H_1) \prod_{i=1,i\neq k}^{n} p(Y_{t,i}, u_i^t|u_0^{t-1}, H_1)] \tag{3.14}$$

Integrating over $Y_{t,1}, Y_{t,2}, ..., Y_{t,k-1}, Y_{t,k+1}, ..., Y_{t,n}$ and unconditioning on u_0^{t-1}, we rewrite (3.14) as

$$J^1(\Gamma^t) = \sum_{u_0^{t-1}} \int_{Y_{t,k}} p(u_k^t = 1|u_0^{t-1}, Y_{t,k}) \sum_{U_k^t} f(U_k^t)$$
$$\times [C_f p(Y_{t,k}|H_0) \prod_{i=1,i\neq k}^{n} p(u_i^t, u_0^{t-1}|H_0) - C_d p(Y_{t,k}|H_1)$$
$$\times [\prod_{i=1,i\neq k}^{n} p(u_i^t, u_0^{t-1}|H_1)] \tag{3.15}$$

To minimize the cost function given in (3.15), we choose

$$p(u_k^t = 1|u_0^{t-1}, Y_{t,k}) = \begin{cases} 1 & if\ A_0 < A_1 \\ 0 & otherwise \end{cases} \tag{3.16}$$

where

$$A_1 = \sum_{U_k^t} f(U_k^t) C_d p(Y_{t,k}|H_1) \prod_{i=1,i\neq k}^{n} p(u_i^t, u_0^{t-1}|H_1)$$

$$A_0 = \sum_{U_k^t} f(U_k^t) C_f p(Y_{t,k}|H_0) \prod_{i=1, i\neq k}^{n} p(u_i^t, u_0^{t-1}|H_0)$$

The k^{th} detector decision rule therefore is given by rewriting (3.16) as:

$$\gamma_k^t(Y_{t,k}, u_0^{t-1}) = u_k^t = \begin{cases} 1 & if\ \dfrac{p(Y_{t,k}|H_1)}{p(Y_{t,k}|H_0)} > \eta_k^t(u_0^{t-1}) \\ 0 & otherwise \end{cases}$$

where $\eta_k^t(u_0^{t-1})$ is the threshold of the k^{th} detector at time step t defined as:

$$\eta_k^t(u_0^{t-1}) = \frac{C_f \sum_{U_k^t} f(U_k^t) p(U_k^t, u_0^{t-1}|H_0)}{C_d \sum_{U_k^t} f(U_k^t) p(U_k^t, u_0^{t-1}|H_1)} \tag{3.17}$$

as stated in Equation (3.5) of Theorem 3.1.

Q.E.D.

It is important to observe that the local decision rule is still a likelihood ratio test. Time step t=1 represents the case without feedback. At this step, the fusion rule has the same form as given in Theorem 3.1. However, the local decision rule is a likelihood ratio test given by Equation (2.18) and the threshold of the test is given by Equation (2.19). For time steps $t > 1$, the threshold $\eta^t(u_0^{t-1})$ of the k^{th} detector is a function of the previous global decision u_0^{t-1} as shown in Equation (3.17). Since the previous global decision u_0^{t-1} takes two values in the case of binary hypothesis testing problem, two thresholds exist for the likelihood ratio test at the local detectors.

System Performance

The system performance will again be given in terms of the system probability of erorr $p_{e_0}^t$. The derivations are the same as in Chapter 2 and, therefore, we only list the results here. The system probability of error $p_{e_0}^t$ is given by

$$p_{e_0}^t = p_{f_0}^t p(H_0) + p_{m_0}^t p(H_1) \tag{3.18}$$

where $p^t_{m_0}$ and $p^t_{f_0}$ are the probability of system miss and false alarm respectively and are given by

$$p^t_{m_0} = p^{t-1}_{m_0}(p^t_{m_0}(u^{t-1}_0 = 0) - p^t_{m_0}(u^{t-1}_0 = 1)) + p^t_{m_0}(u^{t-1}_0 = 1) \tag{3.19}$$

$$p^t_{f_0} = p^{t-1}_{f_0}(p^t_{f_0}(u^{t-1}_0 = 1) - p^t_{f_0}(u^{t-1}_0 = 0)) + p^t_{f_0}(u^{t-1}_0 = 0). \tag{3.20}$$

At time step t=1, the system probability of error equation is the same as Equation (2.27) of Chapter 2.

Next, we compare the performance of the PBPO decentralized detection system with feedback considered here to the conventional PBPO decentralized detection system without feedback shown in Figure 3.2. Intuitively, we expect the system with feedback to perform better because of the additional information available at the local detectors due to feedback. Let the two systems shown in Figures 3.1 and 3.2 be denoted by system A and system B respectively. The performance of system A at time t can be compared with the performance of system B in a meaningful manner if system B processes t observations at each local detector so that the total number of observations processed by system B is also $t \times n$. For the sake of clarity, we only consider the case where all the local detector thresholds are equal to each other. The general problem of nonidentical thresholds may be considered in a similar manner. The result is presented next.

THEOREM 3.3

Consider the PBPO decentralized detection systems A and B shown in Figures 3.1 and 3.2 respectively. The probability of error attained by system A at time t is equal to or less than that attained by system B, i.e.,

$$(p^t_{e_0})_A \leq (p^t_e)_B,$$

under the condition

$$p_{f_0}^{t-1} \leq 1/2$$

where $p_{f_0}^{t-1}$ is the probability of false alarm of system A at time t-1.

Proof:

First, we establish that for a specific non-optimal choice of thresholds at the local detectors in system A, the probability of error is the same as that of system B. It will then follow that with an optimal choice of local thresholds, system A will perform at least as well as system B. We consider the probability of miss and false alarm for system A given by Equations (3.19) and (3.20) and rewrite them in a slightly different form as:

$$p_{m_0}^t = p_{m_0}^t(u_0^{t-1} = 0)p_{m_0}^{t-1} + p_{m_0}^t(u_0^{t-1} = 1)(1 - p_{m_0}^{t-1}) \tag{3.21}$$

$$p_{f_0}^t = p_{f_0}^t(u_0^{t-1} = 1)p_{f_0}^{t-1} + p_{f_0}^t(u_0^{t-1} = 0)(1 - p_{f_0}^{t-1}). \tag{3.22}$$

Recalling Equations (2.24) and (2.26),

$$p_{m_0}^t(u_0^{t-1} = i) = \sum_{U^t} p(u_0^t = 0|U^t)p(U^t|u_0^{t-1} = i, H_1) \tag{3.23}$$

$$p_{f_0}^t(u_0^{t-1} = i) = \sum_{U^t} p(u_0^t = 1|U^t)p(U^t|u_0^{t-1} = i, H_0). \tag{3.24}$$

It is seen that there are two values for $p_{m_0}^t(u_0^{t-1} = i)$ and $p_{f_0}^t(u_0^{t-1} = i)$ corresponding to i=0,1. Also, recall that there are two thresholds at each of the local detectors in system A.

Let η_B^t denote the optimal value of the local threshold (single threshold) for system B. We let the local threshold $\eta_A^t(u_0^{t-1} = 1)$ at each of the detectors in system A take a value less than η_B^t. Since the local detector thresholds have been

assumed to be identical, each threshold value at a local detector corresponds to a specific value of the system probability of miss and false alarm. We let

$$p^t_{m_0}(u_0^{t-1} = 1) = \gamma \tag{3.25}$$

$$p^t_{f_0}(u_0^{t-1} = 1) = \alpha. \tag{3.26}$$

We choose $\eta^t_A(u_0^{t-1} = 0)$ ($> \eta^t_A(u_0^{t-1} = 1)$) at some value such that the following hold:

$$p^t_{m_0}(u_0^{t-1} = 0) = \gamma + \Delta \tag{3.27}$$

$$p^t_{f_0}(u_0^{t-1} = 0) = \alpha - \Delta \tag{3.28}$$

where Δ is any value such that the threshold $\eta^t_A(u_0^{t-1} = 0)$ satisfies both Equations (3.27) and (3.28). Substituting the results of (3.25) and (3.27) in Equation (3.21), the probability of miss for system A is given by

$$\begin{aligned} p^t_{m_0} &= (\gamma + \Delta)p^{t-1}_{m_0} + \gamma(1 - p^{t-1}_{m_0}) \\ &= \gamma p^{t-1}_{m_0} + \Delta p^{t-1}_{m_0} + \gamma(1 - p^{t-1}_{m_0}) \\ p^t_{m_0} &= \gamma + \Delta p^{t-1}_{m_0}. \end{aligned} \tag{3.29}$$

Similarly, the results of (3.26) and (3.28) are substituted in Equation (3.22) to obtain the probability of false alarm for system A as follows:

$$\begin{aligned} p^t_{f_0} &= \alpha p^{t-1}_{f_0} + (\alpha - \Delta)(1 - p^{t-1}_{f_0}) \\ p^t_{f_0} &= \alpha - \Delta(1 - p^{t-1}_{f_0}). \end{aligned} \tag{3.30}$$

Therefore, the system probability of error for system A can be written as:

$$(p^t_{e_0})_A = (\gamma + \Delta p^{t-1}_{m_0})p(H_1) + (\alpha - \Delta(1 - p^{t-1}_{f_0}))p(H_0).$$

Expanding the terms and rearranging,

$$(p^t_{e_0})_A = \gamma p(H_1) + \alpha p(H_0) - \Delta[(1 - p^{t-1}_{f_0})p(H_0) - p^{t-1}_{m_0}p(H_1)]. \tag{3.31}$$

Next, we calculate the probability of error for system B. It is seen that the probability of miss and false alarm for system B can be written in terms of γ and α as follows:

$$p_f = \alpha - a\Delta$$

$$p_m = \gamma + b\Delta$$

where a and b are some real numbers such that $0 < b \leq a < 1$. Without loss of generality, we assume that

$$a + b \leq 1.$$

If $a + b > 1$, then we can redefine α, γ, a and b with respect to the threshold $\eta_A^t(u_0^{t-1} = 0)$ (as opposed to the threshold $\eta_A^t(u_0^{t-1} = 1)$). This will ensure that the assumption $a + b \leq 1$ is satisfied. The probability of error for system B can now be expressed as

$$\begin{aligned}(p_e)_B = \quad & (\alpha - a\Delta)p(H_0) + (\gamma + b\Delta)p(H_1) \\ & = \alpha p(H_0) + \gamma p(H_1) - \Delta[ap(H_0) - bp(H_1)]. \end{aligned} \tag{3.32}$$

From Equations (3.31) and (3.32), we observe that for $\Delta = 0$, the probability of error for system A is the same as that of system B, i.e.,

$$(p_{e_0}^t)_A = (p_e^t)_B.$$

In addition, local thresholds for both systems are the same, i.e.,

$$\eta_A^t(u_0^{t-1} = 1) = \eta_A^t(u_0^{t-1} = 0) = \eta_B^t$$

In other words, the decentralized detection system with feedback and memory reduces to the conventional decentralized detection system B when a sub optimal choice of local thresholds for system A is made as described above.

Next we consider the case $\Delta > 0$ and show that system A performs better than system B when $p_{f_0}^{t-1} < 1/2$. We observe that the first two terms of (3.32) are the

same as the first two terms in (3.31). It remains to be shown that $(ap(H_0)-bp(H_1))$ of (3.32) is less than $[(1-p_{f_0}^{t-1})p(H_0)-p_{m_0}^{t-1}p(H_1)]$ of (3.31). Hence, we have to show that

$$ap(H_0)-bp(H_1)\leq(1-p_{f_0}^{t-1})p(H_0)-p_{m_0}^{t-1}p(H_1).$$

Since we have assumed that $a\leq 1-b$, the following holds

$$(1-b)p(H_0)-bp(H_1)\leq ap(H_0)-bp(H_1)$$

Thus, we need to show that

$$(1-b)p(H_0)-bp(H_1)\leq(1-p_{f_0}^{t-1})p(H_0)-p_{m_0}^{t-1}p(H_1)$$

or,

$$-bp(H_0)-bp(H_1)\leq -p_{f_0}^{t-1}p(H_0)-p_{m_0}^{t-1}p(H_1).$$

This reduces to

$$b\geq p_{f_0}^{t-1}p(H_0)+p_{m_0}^{t-1}p(H_1) \tag{3.33}$$

or,

$$b\geq p_{e_0}^{t-1}.$$

Since $a+b\leq 1$ and $b<a$, $b\leq\frac{1}{2}$. Therefore, the above expression can also be expressed as

$$\frac{1}{2}\geq p_{e_0}^{t-1}. \tag{3.34}$$

Equation (3.34) represents the condition under which system A performs better than system B. Furthermore, a stricter inequality can be obtained by using the convexity of the Receiver Operating Characteristic (ROC), i.e.,

$$\frac{p_{m_0}^{t-1}}{p_{f_0}^{t-1}}\leq 1.$$

This stricter inequality is given by

$$1/2\geq p_{f_0}^{t-1}p(H_0)+p_{f_0}^{t-1}p(H_1) \tag{3.35}$$

or,

$$1/2 \geq p_{f_0}^{t-1}. \tag{3.36}$$

Q.E.D.

Example 3.1

We pursue the same problem as considered in Example 2.1 for the system with memory. Hence, the system consists of two detectors and a fusion center. The input observations at each local detector are also assumed to have a Rayleigh distribution. A priori probabilities are assumed to be equal and minimum probability of error cost assignment is used. For simplicity the SNR at the two detectors are assumed to be equal. For the OR fusion rule, we plot the threshold values $\eta_k^t(u_0^{t-1} = 1)$ and $\eta_k^t(u_0^{t-1} = 0)$ vs. SNR in Figures 3.3 and 3.4 respectively. The probability of system error $p_{e_0}^t$ vs. SNR is plotted in Figures 3.5. The probability of error of a decentralized detection system without feedback vs. SNR is plotted in Figure 3.6. Similarly, for the AND fusion rule, we plot the threshold values $\eta_k^t(u_0^{t-1} = 1)$ and $\eta_k^t(u_0^{t-1} = 0)$ vs. SNR in Figures 3.7 and 3.8 respectively. The probability of system error $p_{e_0}^t$ vs. SNR is plotted in Figure 3.9. The probability of error of the decentralized detection system without feedback vs. SNR is plotted in Figure 3.10.

The plot in Figure 3.3 shows that the threshold $\eta_k^t(u_0^{t-1} = 1)$ decreases as a function of time t. The plot in Figure 3.4 shows that the threshold $\eta_k^t(u_0^{t-1} = 0)$ increases as a function of time and as a function of SNR. Therefore, as time t goes to infinity, the threshold $\eta_k^t(u_0^{t-1} = 1)$ goes to zero and $\eta_k^t(u_0^{t-1} = 0)$ goes to infinity. The plot in Figure 3.5 shows that the probability of system error $p_{e_0}^t$ decreases as a function of time and as a function of SNR as expected. In addition, we observe that the decentralized detection system without feedback has the same

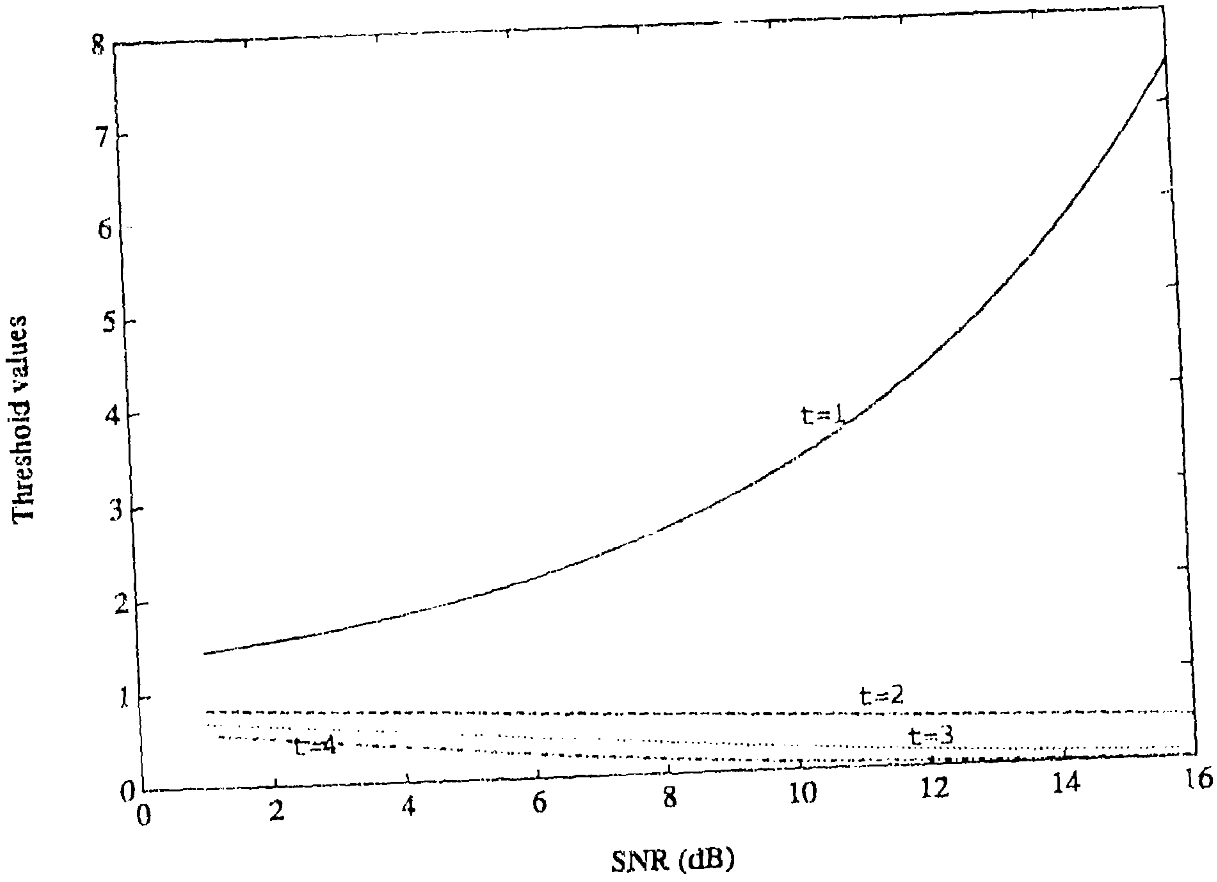

Fig. 3.3: Threshold values given that $u_0^{t-1} = 1$, OR rule.

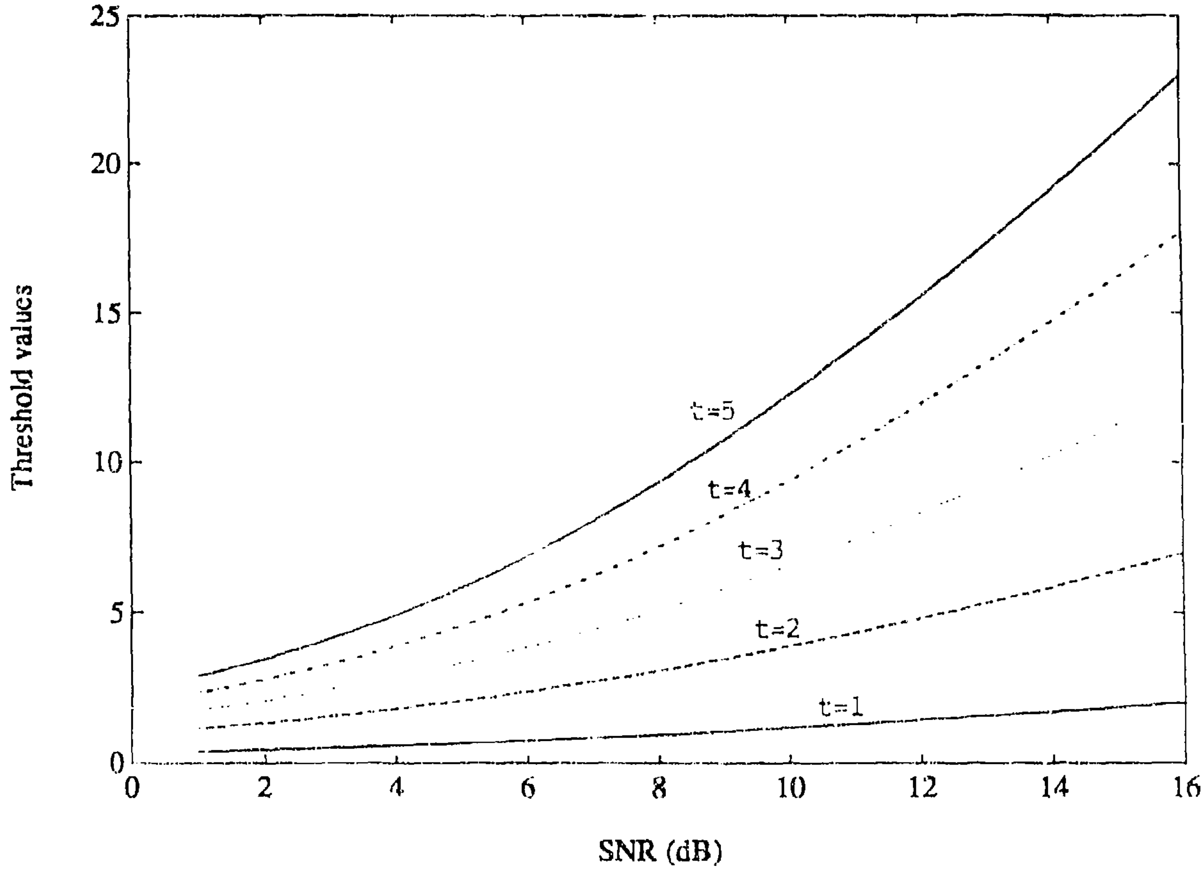

Figure 3.4: Log. of the threshold values given that $u_0^{t-1} = 0$, OR rule.

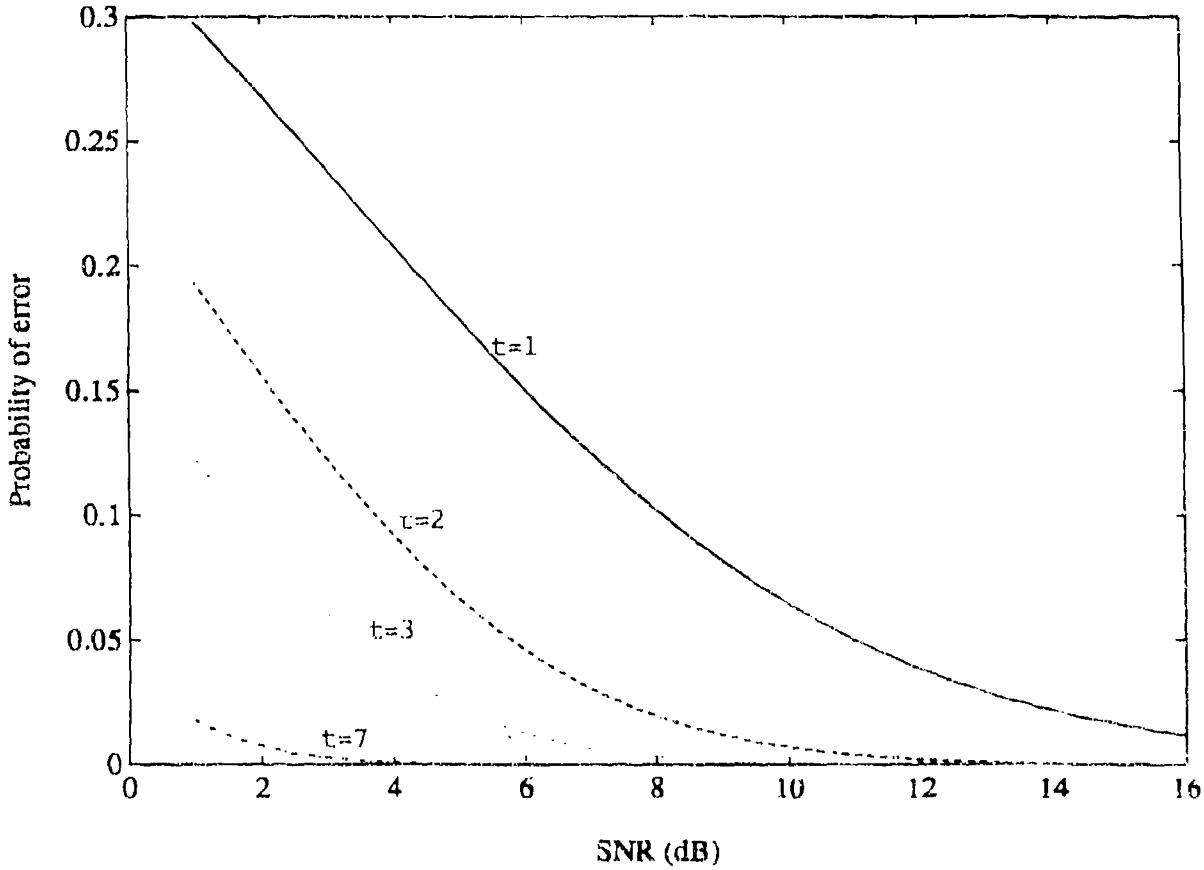

Figure 3.5: Probability of error for the system with memory, OR rule.

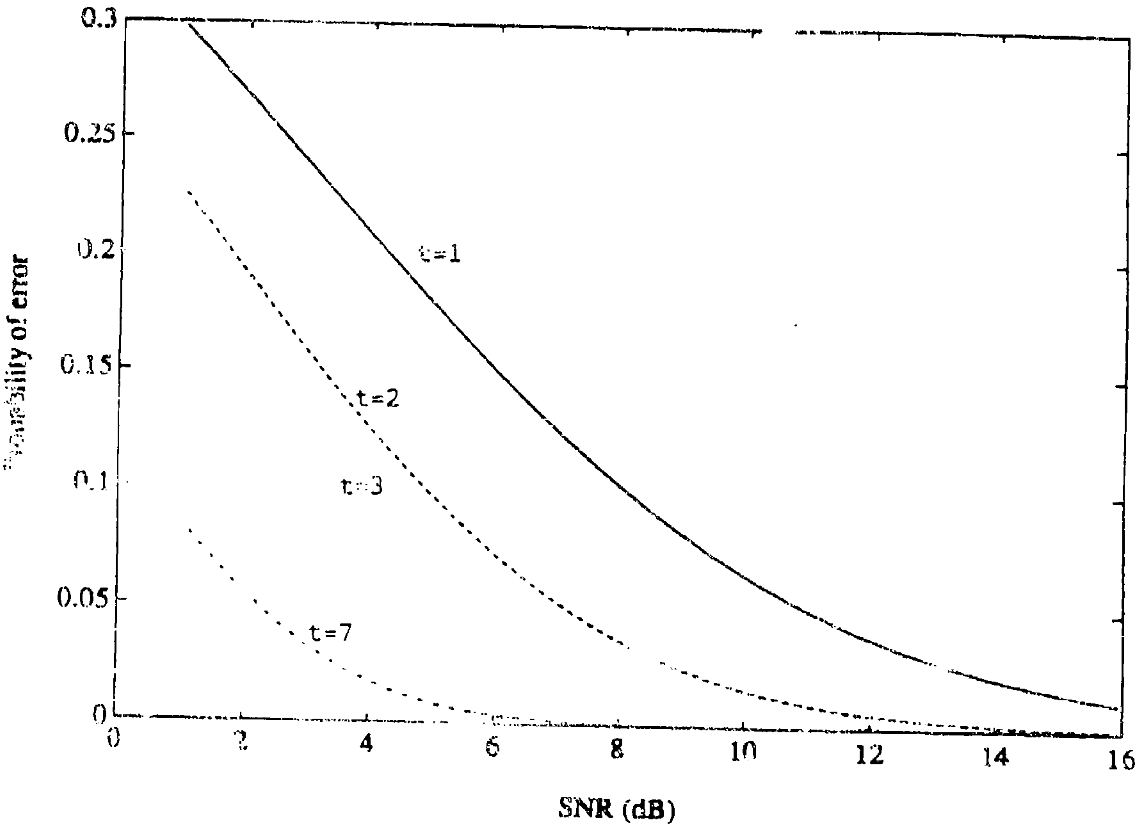

Figure 3.6: Probability of error of the conventional system, OR rule.

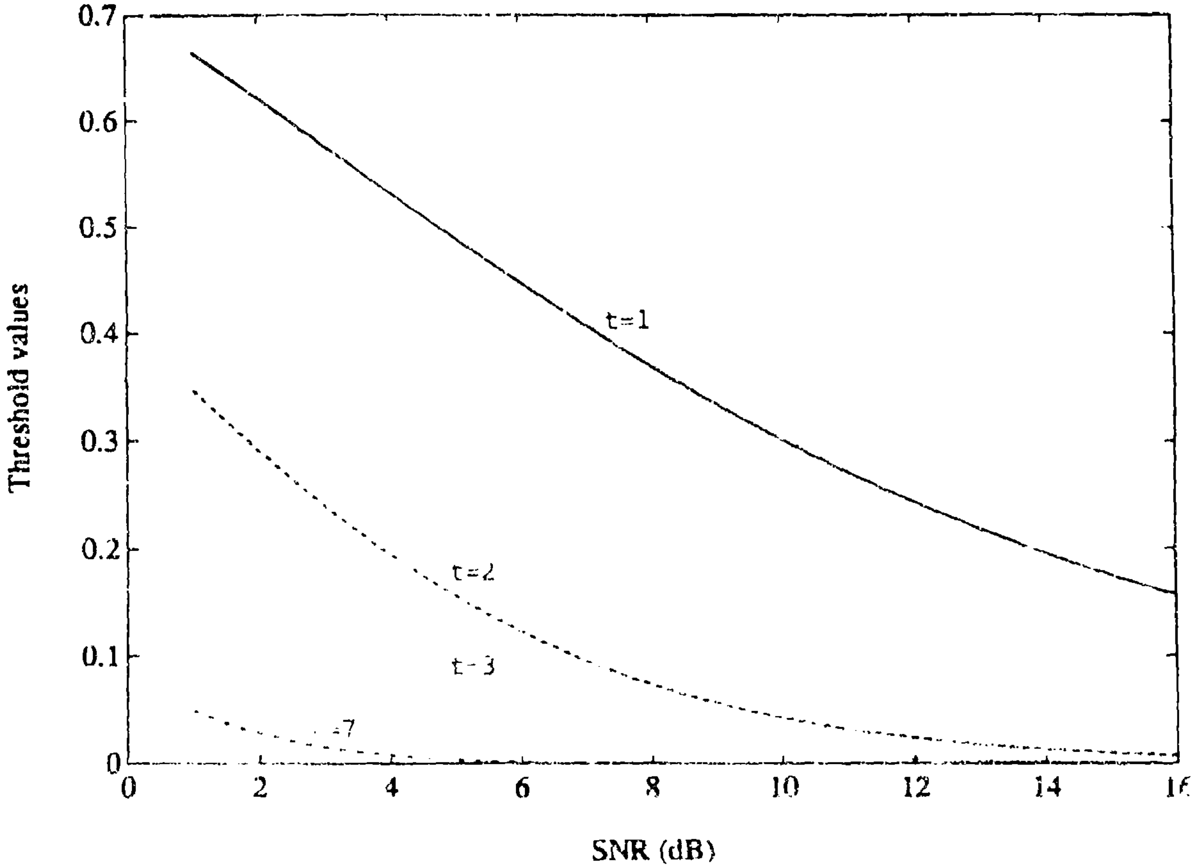

Figure 3.7: Threshold values given that $a_0^{t-1} = 1$, AND rule

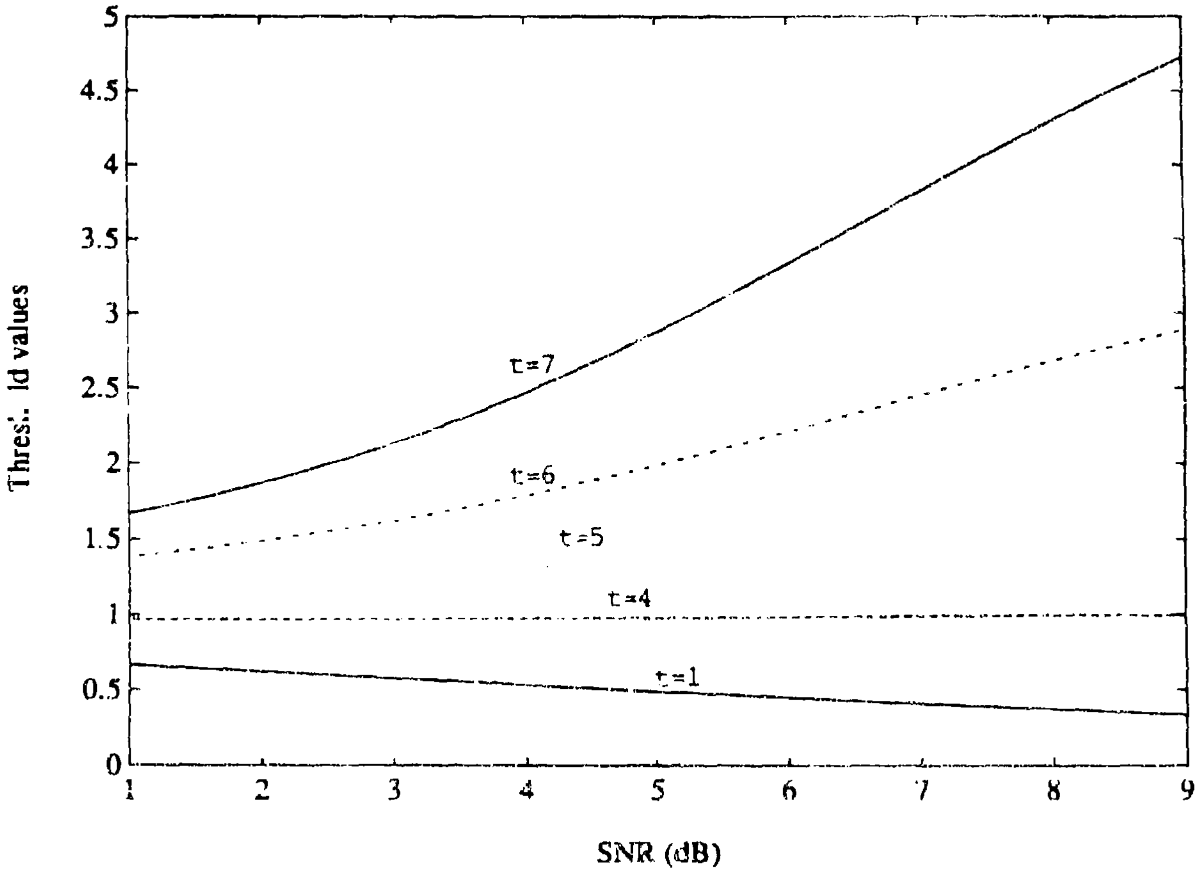

Figure 3.8: Threshold values given that $u_0^{t-1} = 0$, AND rule

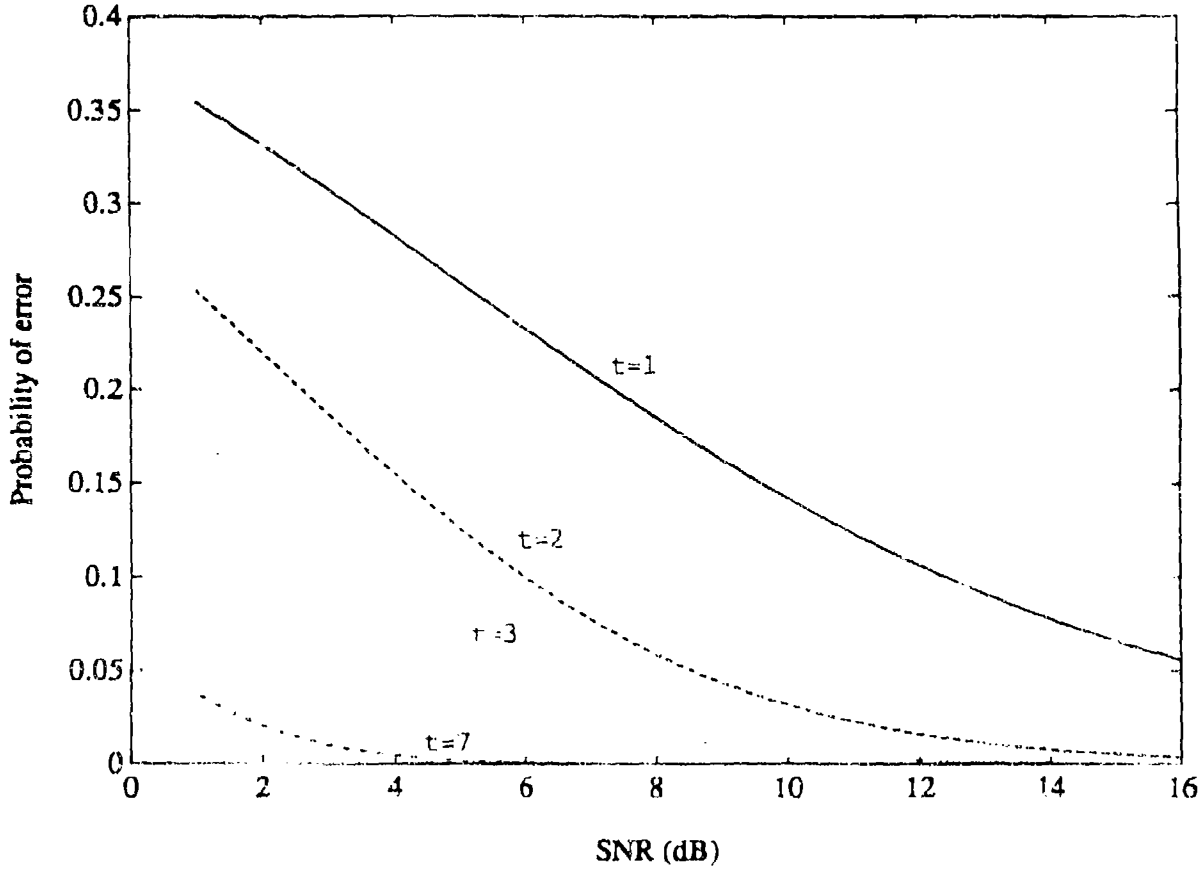

Figure 3.9: Probability of error of the system with feedback, AND rule.

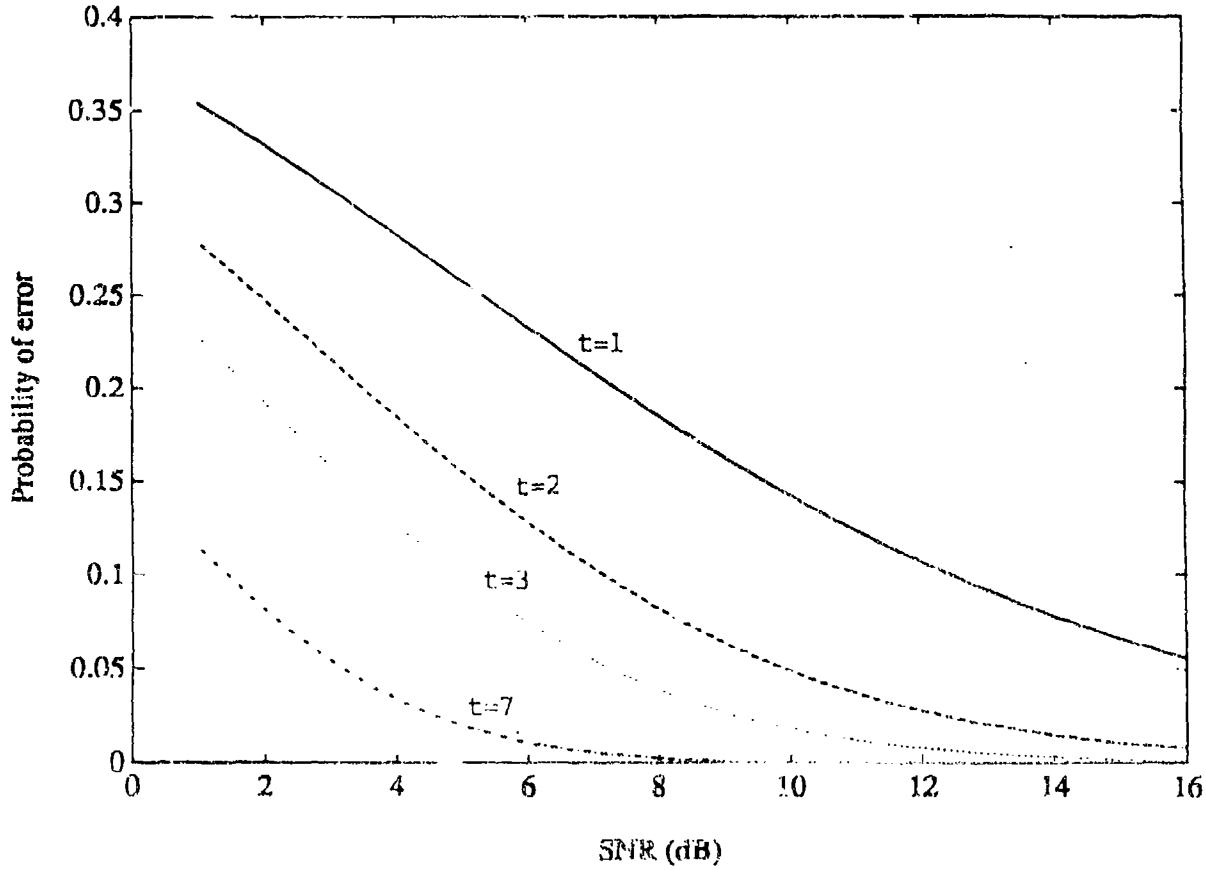

Figure 3.10: Probability of error of the conventional system, AND rule.

probability of error as the system with feedback when one sample is processed at each local detector. As the number of samples per detector increases, it is seen that the decentralized detection system with feedback and memory (Fig. 3.5) has a lower probability of error than that of the conventional decentralized detection system (Fig. 3.6). Furthermore, $p^t_{e_0}$ goes to zero as SNR increases to infinity and as time step t goes to infinity . The plots corresponding to the AND fusion rule shown in Figures 3.7, 3.8 and 3.9 follow a similar behavior as those for the OR rule.

Asymptotic Results

In the above numerical example, we observed that as t increases, the system probability of error decreases. It would be of interest to examine the asymptotic behavior of the probability of error. This result is presented next.

THEOREM 3.4

For the distributed detection system with feedback and memory shown in Figure 3.1, the system probability of error goes to zero as the number of time steps t goes to infinity, i.e.,

$$lim_{t\to\infty}\, p^t_{e_0} = 0.$$

Proof:

We call upon the asymptotic bounds on performance discussed in Blahut [35]. For the hypothesis testing problem, it has been shown that as the number of independent identically distributed measurements, n, goes to infinity, the probability of miss and false alarm go to zero, i.e.,

$$lim_{n\to\infty} p_m = 0$$

$$lim_{n\to\infty} p_f = 0.$$

With these two results in mind, the probabilities of miss and false alarm of a local detector in a decentralized detection system without feedback both go to zero. The probability of miss and false alarm of the decentralized detection system without feedback is given by

$$p_m^t = \sum_{U^t} p(u_0^t = 0|U^t) \prod_{i \in S_0} p_{m_i}^t \prod_{i \in S_1} p_{d_i}^t \tag{3.37}$$

$$p_f^t = \sum_{U^t} p(u_0^t = 1|U^t) \prod_{i \in S_0} (1 - p_{f_i}^t) \prod_{i \in S_1} p_{f_i}^t \tag{3.38}$$

where

S_0: the set of detectors deciding 0 in U^t.

S_1: the set of detectors deciding 1 in U^t.

Notice that Equation (3.37) is a summation over all possible values of U^t. When U^t consists of all 1's, we assume that $p(u_0^t = 0|U^t) = 0$ for any reasonable fusion rule. For all other values of U^t, at least one $p_{m_i}^t$ will exist which goes to zero as t goes to infinity. Thus,

$$lim_{t \to \infty} p_m^t = 0.$$

In a similar fashion, the probability of false alarm goes to zero as well. Using the results of Theorem 3.3 (i.e., the system probability of error of the decentralized detection system with feedback and memory is equal to or less than the probability of error of the decentralized detection system without feedback), the following hold,

$$lim_{t \to \infty} p_{m_0}^t = 0$$

$$lim_{t \to \infty} p_{f_0}^t = 0.$$

Therefore,

$$lim_{t \to \infty} p_{e_0}^t = 0.$$

Q.E.D.

Substituting the results of the above theorem in the threshold Equation (3.5) for $u_0^{t-1} = 0, 1$ it can be shown that the following properties hold,

$$lim_{t \to \infty} \, \eta_k^t(u_0^{t-1} = 0) \to \infty$$

$$lim_{t \to \infty} \, \eta_k^t(u_0^{t-1} = 1) \to 0.$$

Therefore,

$$lim_{t \to \infty} p_{f_0}^t(u^{t-1} = 0) = 0$$

$$lim_{t \to \infty} p_{f_0}^t(u^{t-1} = 1) = 1$$

$$lim_{t \to \infty} p_{m_0}^t(u^{t-1} = 0) = 1$$

$$lim_{t \to \infty} p_{m_0}^t(u^{t-1} = 1) = 0.$$

The probability of detection $p_{d_0}^t(u^{t-1} = i)$, i=0,1, can be obtained from the above properties in a straightforward manner.

The performance advantage exhibited by the decentralized detection system with feedback and memory is useful in many practical situations. Unfortunately increased communication between local detectors and the global decision maker becomes necessary. Therefore, it is desirable to use some communication protocols to reduce the transmission of decisions between local detectors and the global decision maker. In the next section, we propose and analyze two protocols to reduce data transmission.

3.4 Data Transmission Protocols

An important issue to be addressed for the decentralized detection system with feedback is the data transmission requirements. In this system, n decisions are transmitted from the local detectors to the fusion center. The global decision is transmitted from the fusion center to n local detectors. Thus, without any protocols, there are a total of 2n transmissions at each time step. Therefore, the total

number of decision transmissions upto and including time t is 2n×t which is more than the data transmission requirement of the decentralized detection system without feedback. It would be desirable to reduce the data transmission requirements. The metric that we employ is the number of decision transmissions in the system. In this section, we propose two protocols to reduce the average number of decision transmissions. This reduction will result in savings of system resources such as power and bandwidth etc. in a point-to-point communication environment. In communication networking environment, this savings will result in a lower amount of traffic yielding smaller delays and higher information throughputs. In both of the proposed protocols, each detector k, k=0,1,...,n needs to store its previous decision. We denote the number of decision transmissions on forward links and feedback links at time step t by L_f^t and L_b^t respectively. Next, we consider the two protocols individually.

3.4.1 Protocol 1:

In this protocol, at any time step t the global decision maker communicates its decision to all local detectors that disagree with it. Therefore, decision transmission on a feedback link takes place only when the global decision maker disagrees with the local detector corresponding to that feedback link. Thus, we have

Transmit global decision at time t to local detector k if

$$\{u_0^t \neq u_k^t, k = 1, 2, \ldots, n\}.$$

For the forward links, local decision u_k^t is transmitted only if it disagrees with the previous global decision u_0^{t-1}. Therefore,

Transmit local decision at time t from local detector k if

$$\{u_k^t \neq u_0^{t-1}, k = 1, 2, \ldots, n\}.$$

Next, we examine the reduction in the number of decision transmissions achieved

when this protocol is employed. The number of forward decision transmissions at time t given the hypothesis H_i can be expressed as:

$$L_f^t(i) = \sum_{k=1}^{n} I(u_k^t \neq u_0^{t-1} | H_i)$$

and the number of feedback decision transmissions at time t given the hypothesis H_i is:

$$L_b^t(i) = \sum_{k=1}^{n} I(u_0^t \neq u_k^t | H_i)$$

where I(.) is the indicator function given as:

$$I(.) = \begin{cases} 1 & \text{if (.) true} \\ 0 & \text{otherwise} \end{cases}$$

We observe that both L_f^t and L_b^t are discrete random variables taking values in [0,n].

The average number of decision transmissions under the hypothesis H_i is found by taking the expectation of the above random variables. Therefore,

Average number of forward decision transmissions at time t:

$$E\{L_f^t(i)\} = \sum_{k=1}^{n} E\{I(u_k^t \neq u_0^{t-1} | H_i)\}$$

Average number of feedback decision transmissions at time t:

$$E\{L_b^t(i)\} = \sum_{k=1}^{n} E\{I(u_0^t \neq u_k^t | H_i)\}$$

Next, we present Theorem 3.5 where the average number of decision transmissions under this protocol is expressed in terms of the system parameters.

THEOREM 3.5

Consider the decentralized detection system with feedback and memory consisting of n local detectors and a fusion center employing Protocol 1. The average number of decision transmissions at time t is given by

$$E\{L^t\} = p(H_0)E\{L^t(0)\} + p(H_1)E\{L^t(1)\} \qquad (3.39)$$

where $E\{L^t(1)\}$ is the average number of decision transmissions under the hypothesis H_1 given by,

$$\begin{aligned} E\{L^t(1)\} &= n + p_{m_0}^{t-1} p_{m_0}^t (u_0^{t-1} = 1)(n - 2\textstyle\sum_{k=1}^n p_{m_k}^t (u_0^{t-1} = 0)) \\ &\quad + p_{d_0}^{t-1} p_{d_0}^t (u_0^{t-1} = 1)(2\textstyle\sum_{k=1}^n p_{m_k}^t (u_0^{t-1} = 1) - n) \end{aligned} \tag{3.40}$$

and $E\{L^t(0)\}$ is the average number of decision transmissions under the hypothesis H_0 given by,

$$\begin{aligned} E\{L^t(0)\} = n \quad + \quad & p_{f_0}^{t-1} p_{f_0}^t (u_0^{t-1} = 0)(n - 2\sum_{k=1}^n p_{f_k}^t (u_0^{t-1} = 1)) \\ & + (1 - p_{f_0}^{t-1})(1 - p_{f_0}^t (u_0^{t-1} = 0)) \\ & \times (2\sum_{k=1}^n p_{f_k}^t (u_0^{t-1} = 0) - n) \end{aligned} \tag{3.41}$$

Proof:

Recall that:

$$E\{L_f^t(i)\} = \sum_{k=1}^n E\{I(u_k^t \neq u_0^{t-1} | H_i)\}$$

$$E\{L_b^t(i)\} = \sum_{k=1}^n E\{I(u_0^t \neq u_k^t | H_i)\}$$

These can also be written as:

$$E\{L_f^t(i)\} = \sum_{k=1}^n p(u_k^t \neq u_0^{t-1} | H_i) \tag{3.42}$$

$$E\{L_b^t(i)\} = \sum_{k=1}^n p(u_0^t \neq u_k^t | H_i) \tag{3.43}$$

Now, we proceed to derive the average number of decision transmissions $E\{L_f^t(1)\}$ and $E\{L_b^t(1)\}$ separately as follows:

(I) $E\{L_f^t(1)\}$:

Consider the k^{th} term of the summation (3.42). Writing it explicitly in terms of all possible combinations of u_k^t and u_0^{t-1} such that $u_k^t \neq u_0^{t-1}$. We have,

$$\begin{aligned} p(u_k^t \neq u_0^{t-1} | H_i) = \quad & p(u_k^t = 1, u_0^{t-1} = 0 | H_i) \\ & + p(u_k^t = 0, u_0^{t-1} = 1 | H_i) \end{aligned}$$

Conditioning on u_0^{t-1} and expanding, we have

$$\begin{aligned} p(u_k^t \neq u_0^{t-1}|H_i) = \quad & p(u_k^t = 1|u_0^{t-1} = 0, H_i)p(u_0^{t-1} = 0|H_i) \\ & +p(u_k^t = 0|u_0^{t-1} = 1, H_i)p(u_0^{t-1} = 1|H_i) \end{aligned}$$

Summing this over all detectors we get the average number of decision transmissions on the forward links:

$$\begin{aligned} E\{L_f^t(i)\} = \sum_{k=1}^{n} p(u_k^t \neq u_0^{t-1}|H_i) = \quad & \textstyle\sum_{k=1}^{n} p(u_k^t = 1|u_0^{t-1} = 0, H_i)p(u_0^{t-1} = 0|H_i) \\ & +p(u_k^t = 0|u_0^{t-1} = 1, H_i)p(u_0^{t-1} = 1|H_i) \end{aligned}$$

Letting $H_i = H_1$ and rearranging terms, we get

$$E\{L_f^t(1)\} = p_{m_0}^{t-1} \times [\sum_{k=1}^{n} p_{d_k}^t(u_0^{t-1} = 0)] + p_{d_0}^{t-1} \times [\sum_{k=1}^{n} p_{m_k}^t(u_0^{t-1} = 1)] \qquad (3.44)$$

which completes the first part of the proof.

(II) $E\{L_b^t(1)\}$:

Starting with Equation (3.43), we expand the k^{th} term of the summation by introducing the previous global decision u_0^{t-1} as follows:

$$p(u_0^t \neq u_k^t|H_i) = \sum_{u_0^{t-1}} p(u_0^t \neq u_k^t, u_0^{t-1}|H_i)$$

We write this in terms of all possible decision combinations such that $u_0^t \neq u_k^t$:

$$\begin{aligned} p(u_0^t \neq u_k^t|H_i) = \sum_{u_0^{t-1}} \quad & p(u_0^t = 1, u_k^t = 0, u_0^{t-1}|H_i) \\ & +p(u_0^t = 0, u_k^t = 1, u_0^{t-1}|H_i) \end{aligned}$$

Conditioning on u_0^t and u_0^{t-1} and expanding, we have:

$$\begin{aligned} p(u_0^t \neq u_k^t|H_i) = \quad & \textstyle\sum_{u_0^{t-1}} p(u_k^t = 0|u_0^t = 1, u_0^{t-1}, H_i)p(u_0^t = 1|u_0^{t-1}, H_i)p(u_0^{t-1}|H_i) \\ & +p(u_k^t = 1|u_0^t = 0, u_0^{t-1}, H_i)p(u_0^t = 0|u_0^{t-1}, H_i)p(u_0^{t-1}|H_i) \end{aligned}$$

Rearranging and observing that $u_k^t = i$ conditioned on u_0^{t-1} and H_i is independent of u_0^t, we have

$$\begin{aligned} p(u_0^t \neq u_k^t|H_i) = \sum_{u_0^{t-1}} p(u_0^{t-1}|H_i) \quad & [p(u_k^t = 0|u_0^{t-1}, H_i)p(u_0^t = 1|u_0^{t-1}, H_i) \\ & +p(u_k^t = 1|u_0^{t-1}, H_i)p(u_0^t = 0|u_0^{t-1}, H_i)] \end{aligned}$$

Summing over all detectors and letting $H_i = H_1$ yields the average number of feedback decision transmissions,

$$E\{L_b^t(1)\} = \sum_{u_0^{t-1}} p(u_0^{t-1}|H_1) \quad \sum_{k=1}^{n}[p(u_k^t = 0|u_0^{t-1}, H_1)p(u_0^t = 1|u_0^{t-1}, H_1)$$
$$+p(u_k^t = 1|u_0^{t-1}, H_1)p(u_0^t = 0|u_0^{t-1}, H_1)]$$

This could be rewritten as:

$$E\{L_b^t(1)\} = \sum_{u_0^{t-1}} p(u_0^{t-1}|H_1) \sum_{k=1}^{n}[p_{m_k}^t(u_0^{t-1}) \times p_{d_0}^t(u_0^{t-1}) + p_{d_k}^t(u_0^{t-1}) \times p_{m_0}^t(u_0^{t-1})] \tag{3.45}$$

Substituting $p_{d_k}^t(u_0^{t-1})$ by $1 - p_{m_k}^t(u_0^{t-1})$ and rearranging, we have

$$E\{L_b^t(1)\} = \sum_{u_0^{t-1}} p(u_0^{t-1}|H_1)[n \times p_{m_0}^t(u_0^{t-1}) + (1 - 2p_{m_0}^t(u_0^{t-1})) \sum_{k=1}^{n} p_{m_k}^t(u_0^{t-1})] \tag{3.46}$$

Finally, summing $E\{L_b^t(1)\}$ and $E\{L_f^t(1)\}$ we obtain the result of Equation (3.40). In a similar manner, we may derive the results given in (3.41). Finally, the results of (3.40) and (3.41) can be used to obtain the overall result of Equation (3.39).

Q.E.D.

Next, we consider the asymptotic behavior of the average number of data transmissions under this protocol and present the result in Lemma 3.1.

Lemma 3.1 :

When Protocol 1 is used, the average number of decision transmissions $E\{L^t\}$ for the system under consideration goes to zero as the number of time steps t increases to infinity.

Proof:

We recall the asymptotic properties of the decentralized detection system with memory and feedback from Section 3.3 to prove this Lemma, namely,

$lim_{t\to\infty} p_{m_0}^t = 0.$

$lim_{t\to\infty} p_{d_0}^t = 1.$

$lim_{t\to\infty} p_{m_k}^t(u_0^{t-1} = 1) = 0$, k=1,2,...,n.

$lim_{t\to\infty} p_{d_0}^t(u_0^{t-1} = 1) = 1.$

Using these asymptotic properties in (3.40), we have

$$lim_{t\to\infty} E\{L^t(1)\} = 0$$

Similarly, it can be shown that

$$lim_{t\to\infty} E\{L^t(0)\} = 0$$

Therefore,

$$lim_{t\to\infty} E\{L^t\} = 0$$

Q.E.D.

Next we consider another protocol for the reduction of decision transmissions.

3.4.2 Protocol 2:

In this protocol, at any time step t, the global decision maker communicates its decision to all the local detectors when it disagrees with the previous global decision. Therefore, a feedback decision transmission on all feedback links takes place when the current global decision disagrees with the previous global decision, i.e.,

Transmit global decision at time t to all local detectors if $\{u_0^t \neq u_0^{t-1}\}$.

For the forward links, local decision u_k^t is transmitted on the k^{th} forward link only if it disagrees with the previous local decision u_k^{t-1}. Hence,

Transmit local decision from a local detector k if $\{u_k^t \neq u_k^{t-1}\}$

The reduction in the average number of decision transmissions achieved by employing this protocol is examined next. We express the number of forward

decision transmissions given the hypothesis H_i as:

$$L_f^t(i) = \sum_{k=1}^{n} I(u_k^t \neq u_k^{t-1} | H_i)$$

and the number of feedback decision transmissions given the hypothesis H_i as:

$$L_b^t(i) = \sum_{k=1}^{n} I(u_0^t \neq u_0^{t-1} | H_i)$$

where I(.) is the indicator function defined earlier. We observe that both $L_f^t(i)$ and $L_b^t(i)$ are discrete random variables taking integer values in [0,n].

The average number of decision transmissions given the hypothesis H_i is found by taking the expectation of the above random variables. Therefore,

Average number of forward decision transmissions at time t:

$$E\{L_f^t(i)\} = \sum_{k=1}^{n} E\{I(u_k^t \neq u_k^{t-1} | H_i)\}$$

Average number of feedback decision transmissions at time t:

$$E\{L_b^t(i)\} = \sum_{k=1}^{n} E\{I(u_0^t \neq u_0^{t-1} | H_i)\}$$

Next, the average number of decision transmissions for this protocol is presented in Theorem 3.6.

THEOREM 3.6

Consider the decentralized detection system with feedback and memory consisting of n local detectors and a fusion center employing Protocol 2. The average number of decision transmissions at time t is given by

$$E\{L^t\} = p(H_0)E\{L^t(0)\} + p(H_1)E\{L^t(1)\} \tag{3.47}$$

where $E\{L^t(i)\}$, i=0,1, is the average number of decision transmissions at time t under the hypothesis H_i and given by,

$$E\{L^t(i)\} = E\{L_b^t(i)\} + E\{L_f^t(i)\}$$

$lim_{t\to\infty} p_{m_0}^t = 0.$

$lim_{t\to\infty} p_{d_0}^t = 1.$

$lim_{t\to\infty} p_{m_k}^t(u_0^{t-1} = 1) = 0$, k=1,2,...,n.

$lim_{t\to\infty} p_{d_0}^t(u_0^{t-1} = 1) = 1.$

Using these asymptotic properties in (3.40), we have

$$lim_{t\to\infty} E\{L^t(1)\} = 0$$

Similarly, it can be shown that

$$lim_{t\to\infty} E\{L^t(0)\} = 0$$

Therefore,

$$lim_{t\to\infty} E\{L^t\} = 0$$

Q.E.D.

Next we consider another protocol for the reduction of decision transmissions.

3.4.2 Protocol 2:

In this protocol, at any time step t, the global decision maker communicates its decision to all the local detectors when it disagrees with the previous global decision. Therefore, a feedback decision transmission on all feedback links takes place when the current global decision disagrees with the previous global decision, i.e.,

Transmit global decision at time t to all local detectors if $\{u_0^t \neq u_0^{t-1}\}$.

For the forward links, local decision u_k^t is transmitted on the k^{th} forward link only if it disagrees with the previous local decision u_k^{t-1} . Hence,

Transmit local decision from a local detector k if $\{u_k^t \neq u_k^{t-1}\}$.

The reduction in the average number of decision transmissions achieved by employing this protocol is examined next. We express the number of forward

The average number of feedback decision transmissions $E\{L_b^t(i)\}$,i=0,1, at time t are given by

$$E\{L_b^t(1)\} = n \times [p_{m_0}^t(u_0^{t-1} = 1) \times p_{d_0}^{t-1} + p_{d_0}^t(u_0^{t-1} = 0) \times p_{m_0}^{t-1}] \tag{3.48}$$

$$\begin{aligned} E\{L_b^t(0)\} = n\times \quad & [(1 - p_{f_0}^t(u_0^{t-1} = 1)) \times p_{f_0}^{t-1} \\ & + p_{f_0}^t(u_0^{t-1} = 0) \times (1 - p_{f_0}^{t-1})] \end{aligned} \tag{3.49}$$

The average number of forward decision transmissions $E\{L_f^t(i)\}$, i=0,1, at time t are given by

$$E\{L_f^t(1)\} = \sum_{u_0^{t-1}} \sum_{k=1}^{n} p_{m_k}^t(u_0^{t-1}) \times L_1 + p_{d_k}^t(u_0^{t-1}) \times L_2 \tag{3.50}$$

where $L_1 = \sum_{u_0^{t-2}} p_{d_k}^{t-1}(u_0^{t-2}) \times k1$

$L_2 = \sum_{u_0^{t-2}} p_{m_k}^{t-1}(u_0^{t-2}) \times k1$

$k1 = p(u_0^{t-1}|u_0^{t-2}, H_1) \times p(u_0^{t-2}|H_1)$.

$$E\{L_f^t(0)\} = \sum_{u_0^{t-1}} \sum_{k=1}^{n} p_{f_k}^t(u_0^{t-1}) \times L_3 + (1 - p_{f_k}^t(u_0^{t-1}) \times L_4 \tag{3.51}$$

where $L_3 = \sum_{u_0^{t-2}} (1 - p_{f_k}^{t-1}(u_0^{t-2})) \times k0$

$L_4 = \sum_{u_0^{t-2}} p_{f_k}^{t-1}(u_0^{t-2}) \times k0$

$k0 = p(u_0^{t-1}|u_0^{t-2}, H_0) \times p(u_0^{t-2}|H_0)$.

Proof:

As before,

$$E\{L_f^t(i)\} = \sum_{k=1}^{n} p(u_k^t \neq u_k^{t-1}|H_i) \tag{3.52}$$

$$E\{L_b^t(i)\} = \sum_{k=1}^{n} p(u_0^t \neq u_0^{t-1}|H_i) \tag{3.53}$$

Now, we proceed to derive the average number of decision transmissions for any H_i. The final results are obtained only for the case $H_i = H_1$. The results for the $H_i = H_0$ case can be obtained in a similar fashion. We derive $E\{L_f^t(1)\}$ and $E\{L_b^t(1)\}$ separately as follows:

(I) $E\{L_b^t(1)\}$:

We observe that in Equation (3.53), the summation argument is independent of the summation index k, hence

$$E\{L_b^t(i)\} = n \times p(u_0^t \neq u_0^{t-1}|H_i)$$

Expanding in terms of all possible decision combinations such that $u_0^t \neq u_0^{t-1}$,

$$E\{L_b^t(i)\} = n \times [p(u_0^t = 0, u_0^{t-1} = 1|H_i) + p(u_0^t = 1, u_0^{t-1} = 0|H_i)]$$

Conditioning on u_0^{t-1} and expanding

$$\begin{aligned} E\{L_b^t(i)\} = n\times \quad & [p(u_0^t = 0|u_0^{t-1} = 1, H_i)p(u_0^{t-1} = 1|H_i) \\ & + p(u_0^t = 1|u_0^{t-1} = 0, H_i)p(u_0^{t-1} = 0|H_i)] \end{aligned} \tag{3.54}$$

Substituting $H_i = H_1$ and rewriting in terms of $p_{m_0}^t$ and $p_{d_0}^t$ we get

$$E\{L_b^t(1)\} = n[p_{m_0}^t(u_0^{t-1} = 1) \times p_{d_0}^{t-1} + p_{d_0}^t(u_0^{t-1} = 0) \times p_{m_0}^{t-1}]$$

Similarly, letting $H_i = H_0$ in Equation (3.54), we have

$$E\{L_b^t(0)\} = n[(1 - p_{f_0}^t(u_0^{t-1} = 1)) \times p_{f_0}^{t-1} + p_{f_0}^t(u_0^{t-1} = 0) \times (1 - p_{f_0}^{t-1})]$$

as stated in Equations (3.48) and (3.49).

(II) $E\{L_f^t(1)\}$:

We introduce the previous global decision u_0^{t-1} into Equation (3.52), hence

$$E\{L_f^t(i)\} = \sum_{u_0^{t-1}} \sum_{k=1}^{n} p(u_k^t \neq u_k^{t-1}, u_0^{t-1}|H_i)$$

Expanding in terms of all possible decision combinations such that $u_k^t \neq u_k^{t-1}$

$$\begin{aligned} E\{L_f^t(i)\} = \sum_{u_0^{t-1}} \sum_{k=1}^{n} \quad & p(u_k^t = 0, u_k^{t-1} = 1, u_0^{t-1}|H_i) \\ & + p(u_k^t = 1, u_k^{t-1} = 0, u_0^{t-1}|H_i) \end{aligned} \tag{3.55}$$

Conditioning on u_k^{t-1} and u_0^{t-1} and expanding, we have

$$E\{L_f^t(i)\} = \sum_{u_0^{t-1}} \sum_{k=1}^{n} \quad p(u_k^t = 0 | u_k^{t-1} = 1, u_0^{t-1}, H_i) p(u_k^{t-1} = 1, u_0^{t-1} | H_i)$$
$$+ p(u_k^t = 1 | u_k^{t-1} = 0, u_0^{t-1}, H_i) p(u_k^{t-1} = 0, u_0^{t-1} | H_i) \quad (3.56)$$

Observing that the local decision u_k^t conditioned on u_0^{t-1} and H_i is independent of u_k^{t-1}, we rewrite the above as:

$$E\{L_f^t(i)\} = \sum_{u_0^{t-1}} \sum_{k=1}^{n} \quad p(u_k^t = 0 | u_0^{t-1}, H_i) \times T_1(i)$$
$$+ p(u_k^t = 1 | u_0^{t-1}, H_i) \times T_2(i) \quad (3.57)$$

where $T_1(i)$ and $T_2(i)$ are used for notational convenience as follows

$$T_1(i) = p(u_k^{t-1} = 1, u_0^{t-1} | H_i)$$

$$T_2(i) = p(u_k^{t-1} = 0, u_0^{t-1} | H_i)$$

The terms $T_1(i)$ and $T_2(i)$ cannot be evaluated yet and need further work. Introducing u_0^{t-2} into $T_1(i)$ and conditioning on u_0^{t-1} and u_0^{t-2} , we get

$$T_1(i) = \sum_{u_0^{t-2}} p(u_k^{t-1} = 1 | u_0^{t-1}, u_0^{t-2}, H_i) p(u_0^{t-1}, u_0^{t-2} | H_i)$$

Further conditioning the last term on u_0^{t-2} and observing that u_k^{t-1} conditioned on u_0^{t-2} and H_i is independent of u_0^{t-1}, we have

$$T_1(i) = \sum_{u_0^{t-2}} p(u_k^{t-1} = 1 | u_0^{t-2}, H_i) p(u_0^{t-1} | u_0^{t-2}, H_i) p(u_0^{t-2} | H_i) \quad (3.58)$$

In a similar fashion, $T_2(i)$ is obtained as:

$$T_2(i) = \sum_{u_0^{t-2}} p(u_k^{t-1} = 0 | u_0^{t-2}, H_i) p(u_0^{t-1} | u_0^{t-2}, H_i) p(u_0^{t-2} | H_i) \quad (3.59)$$

For i=1, $T_1(1)$ and $T_2(1)$ can be written as:

$$T_1(1) = \sum_{u_0^{t-2}} P_{d_k}^{t-1}(u_0^{t-2}) \times p(u_0^{t-1} | H_1) p(u_0^{t-2} | H_1) \quad (3.60)$$

$$T_2(1) = \sum_{u_0^{t-2}} p_{m_k}^{t-1}(u_0^{t-2}) \times p(u_0^{t-1}|H_1)p(u_0^{t-2}|H_1) \qquad (3.61)$$

It is seen that $T_1(1)$ and $T_2(1)$ are the same as L_1 and L_2 respectively in Theorem 3.6. Similarly, for i=0, $T_1(0)$ and $T_2(0)$ are the same as L_3 and L_4 in Theorem 3.6. Therefore, the average number of decision transmissions is evaluated by substituting the results of Equations (3.48)-(3.50) in Equation (3.47).

Q.E.D.

The asymptotic behavior of the average number of decision transmissions under Protocol 2 is considered in Lemma 3.2 next.

Lemma 3.2 :

When Protocol 2 is used, the average number of decision transmissions $E\{L^t\}$ for the system under consideration approaches zero as the number of time steps t increases to infinity.

Proof:

Again, we call upon the asymptotic properties as listed in Theorem 3.4, namely

$lim_{t\to\infty} p_{m_0}^t = 0.$

$lim_{t\to\infty} p_{d_0}^t = 1.$

$lim_{t\to\infty} p_{m_k}^t(u_0^{t-1} = 1) = 0$, k=1,2,...,n.

$lim_{t\to\infty} p_{d_0}^t(u_0^{t-1} = 1) = 1.$

Using these properties in Equations (3.48) and (3.49) it can be shown that the average number of decision transmissions given the hypothesis H_1, $E\{L^t(1)\}$ goes to zero as t goes to infinity. In a similar fashion, the average number of decision transmissions given the hypothesis H_0, $E\{L^t(0)\}$ goes to zero as t goes to infinity. Therefore, the average number of decision transmissions for this second protocol $E\{L^t\}$ goes to zero as t goes to infinity.

Q.E.D.

Next, we present a numerical example that shows the behavior of the average number of decision transmissions $E\{L^t\}$ for Protocols 1 and 2.

Example 3.2

We further pursue Example 3.1 and investigate the performance of the protocols. For both Protocols 1 and 2, we plot the average number of decision transmissions $E\{L^t\}$ vs. SNR for the OR and the AND fusion rules. The results given in Theorems 3.5 and 3.6 are used for the computations.

The plots of Figures 3.11 and 3.12 show that for the OR fusion rule, the average number of decision transmissions for both the first and the second protocols decrease as SNR values increase and as time step t increases. From Figures 3.11 and 3.12, it is seen that the average number of decision transmissions for Protocol 1 decreases more rapidly than Protocol 2. The plots of the average number of decision transmissions for Protocols 1 and 2 for the AND fusion rule are given in Figures 3.13 and 3.14 respectively. The average number of transmissions $E\{L^t\}$ is observed to be decreasing again as was the case with the OR fusion rule. It is interesting to note that as t goes to infinity, the average number of decision transmissions goes to zero for both protocols , i.e. no decision transmissions are required on an average.

3.5 Discussion

In this chapter, we have considered a decentralized detection system with feedback and memory. The incorporation of memory at the local detectors provided a considerable enhancement for the system performance. This system was optimized using the Bayesian formulation. Using the PBPO solution methodology, we derived decision rules for the local detectors and the fusion center. The system probability

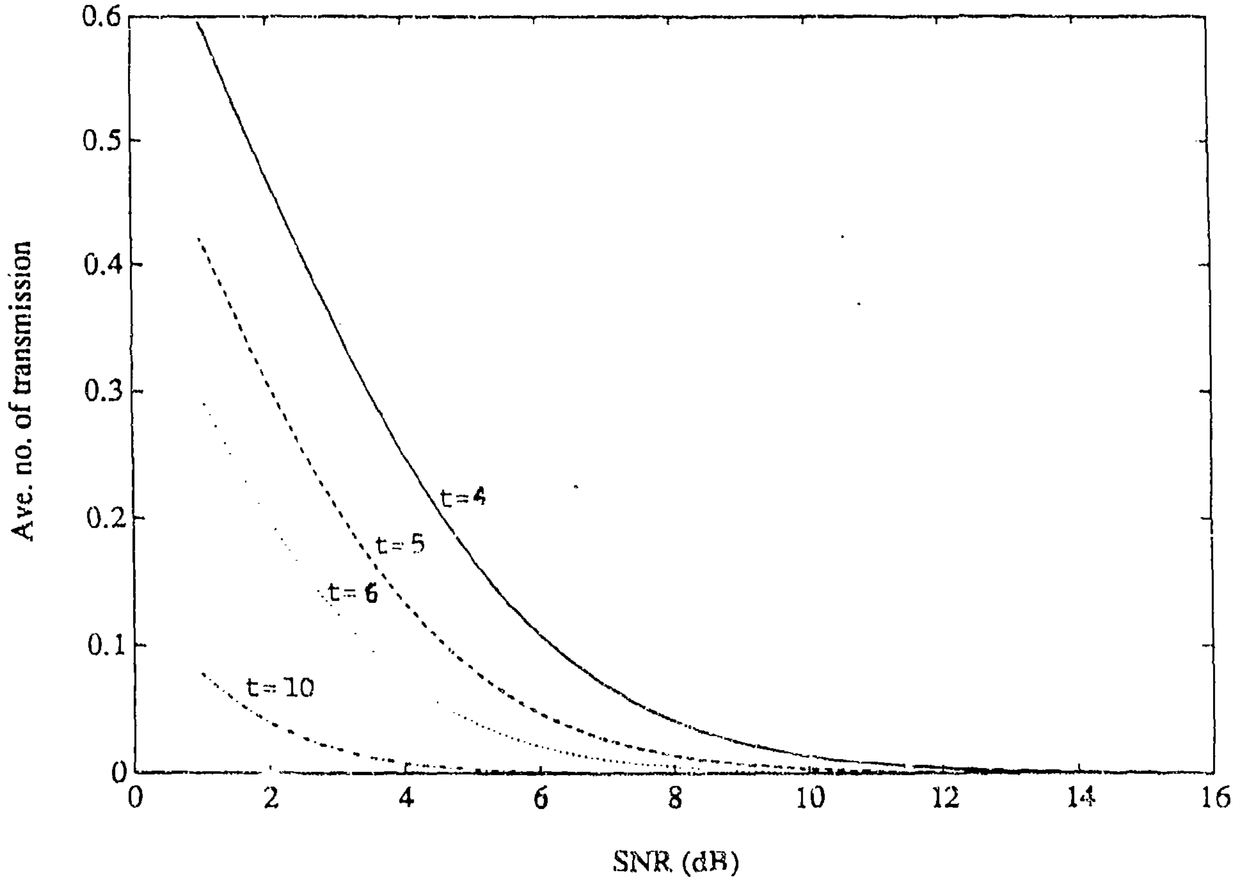

Figure 3.11: Average number of transmissions using Protocol 1, OR rule.

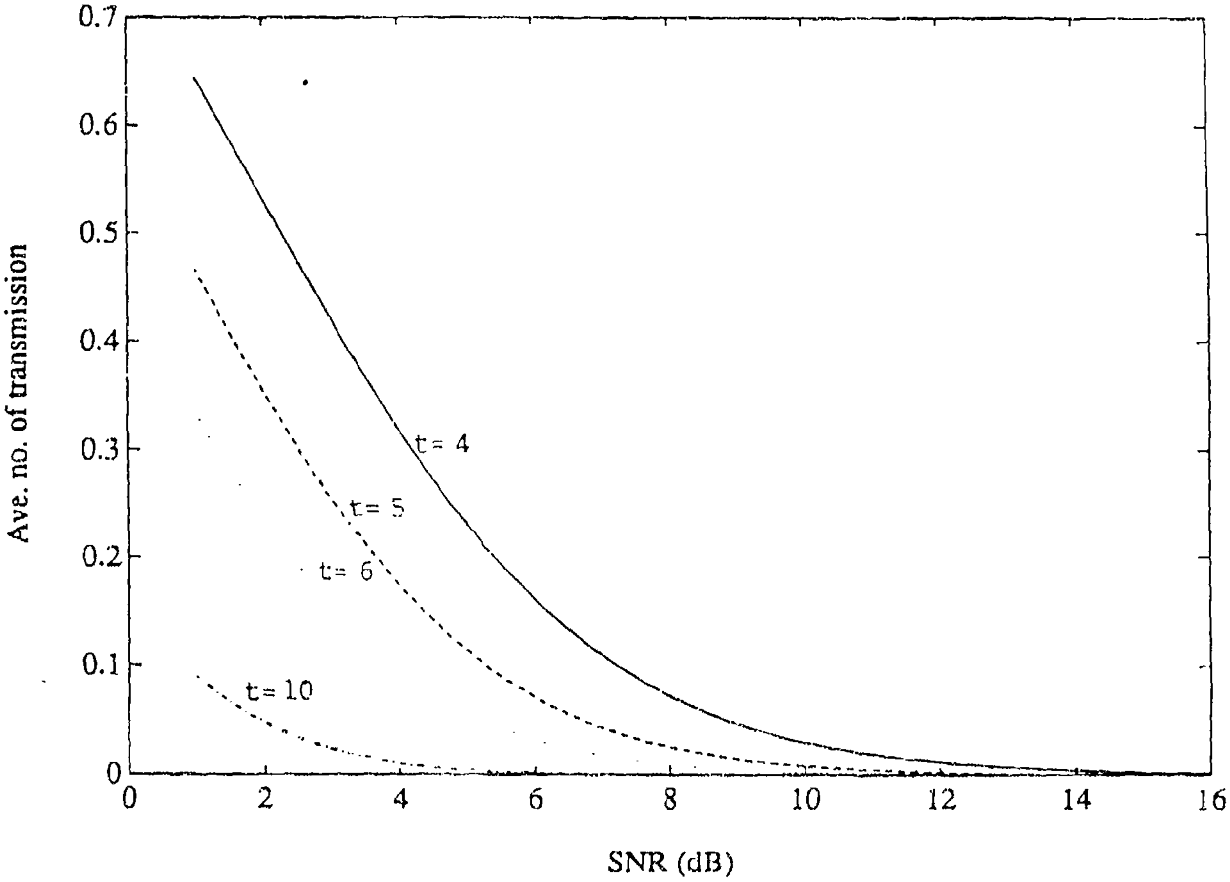

Figure 3.12: Average number of transmissions using Protocol 2, OR rule.

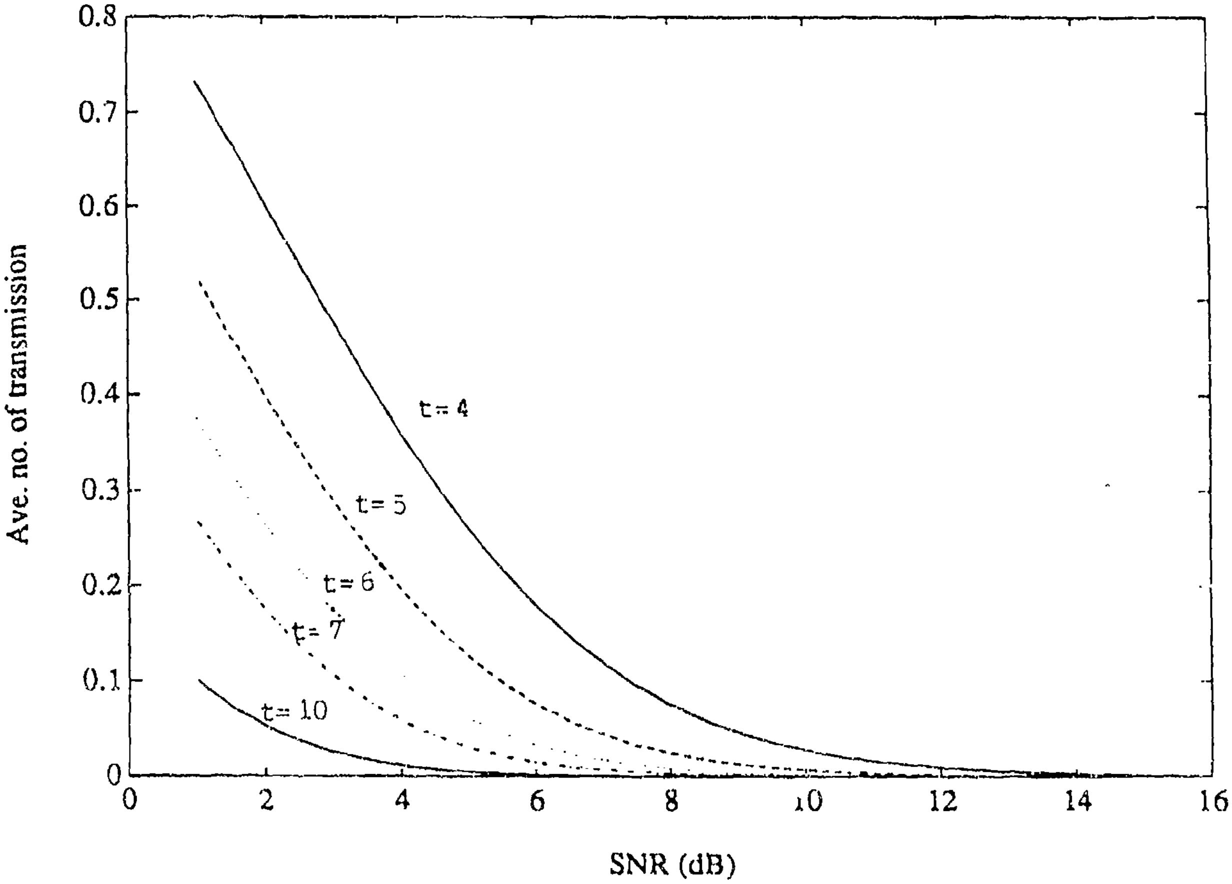

Figure 3.13: Average number of transmissions using Protocol 1, AND rule.

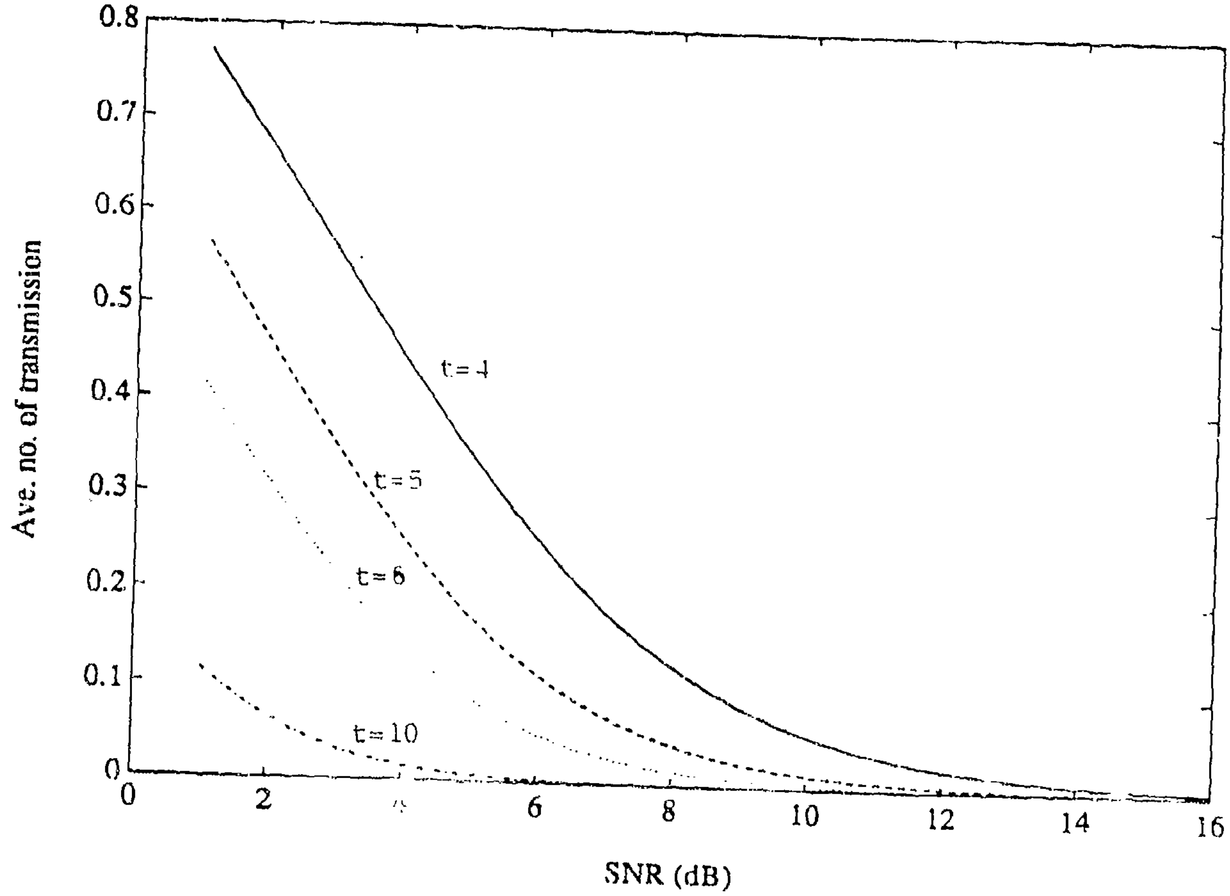

Figure 3.14: Average number of transmissions using Protocol 2, AND rule.

of error was derived and shown to be at least as good as that of the conventional decentralized detection system without feedback. The system probability of error was shown to decreases to zero as the number of observations increases to infinity. An important issue that arises in this system is that of decision transmission. Due to the feedback links, the system is characterized by an increase in decision transmission. We proposed and studied two protocols to reduce decision transmission requirements. The average number of decision transmission was shown to go to zero asymptotically when Protocol 1 or 2 is deployed. Numerical results were obtained for a system of two detectors and a fusion center. Using the OR fusion rule, the decentralized detection system with feedback and memory was shown to have a lower probability of error as compared to the conventional decentralized detection system. Similar results were found for the AND fusion rule.

Chapter 4

A Unified Approach to the Decentralized Detection Problem

4.1 Introduction

In the previous chapters, we have considered the problem of Bayesian hypothesis testing in decentralized detection systems with feedback. Several other decentralized detection network topologies have been investigated in the literature, e.g., the conventional decentralized detection network without feedback, the serial network, the hierarchical network, etc. . In this chapter, we provide a unified representation for different decentralized detection network topologies. This representation is inspired by the definition of information structure given in [22, 23]. This unified representation is then used to obtain PBPO decision rules for various decentralized detection systems.

In Section 4.2, we define the communication structure of organizations as it applies to team decision making. It is shown as to how a number of decentralized

at the detector corresponding to the column k are given by the k^{th} column. We define the decision input of the k^{th} detector as follows:

$$I_k = \{u_i : D_{ik} = 1; \textit{for all } i\} \tag{4.1}$$

Thus, decentralized detection systems with any configuration can be specified in terms of the communication matrix D. Next, we present a couple of examples illustrating the communication structure representation of decentralized detection systems.

Example 4.1:

For a serial system consisting of N detectors with observations y_i at each detector i, i = 1, 2, ..., N(Figure 4.1), the matrix D is given by an off diagonal matrix of dimension N x N as follows:

$$D = \begin{array}{c} \textit{det. no.} \\ 1 \\ 2 \\ 3 \\ \vdots \\ N-1 \\ N \end{array} \begin{array}{c} 1\ 2\ 3\ 4\ \cdots\ N \\ \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \end{array}$$

The entries of the matrix are obtained from the block diagram of the serial system. Detector numbers are also indicated for convenience of the reader. The (i,k) element is one if detector i transmits its decision to detector k. For example, $D_{12} = 1$ indicates that the decision of detector 1 is fed to detector 2. Using Equation (4.1), the decision input of the N^{th} detector is given by,

$$I_N = u_{N-1} \cdot$$

The first column of the D matrix has all zero entries indicating that there is no

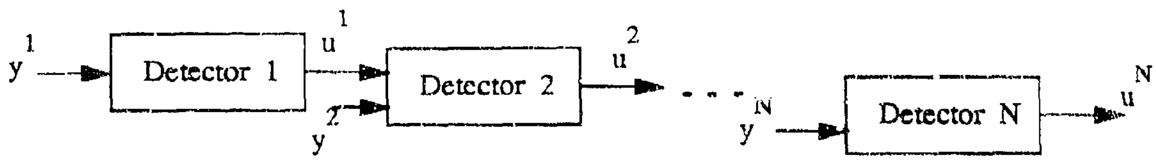

Fig. 4.1: A serial system consisting of N detectors.

decision input, i.e.,

$$I_1 = no\ input$$

Example 4.2:

For a decentralized detection system with a fusion center consisting of n local detectors(Figure 4.2), the communication matrix D is of dimension (n+1)x(n+1) and given by:

$$D = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & \cdots & n & 0 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ \vdots \\ n \\ 0 \end{array} & \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \end{array}$$

Note that the global decision maker is denoted by detector number 0 and it appears in the last row and column of the matrix. As seen form this matrix, there are no decision inputs to the k^{th} detector,
k=1,2,...,n, i.e.,

$$I_k = no\ input$$

However, the column corresponding to detector 0 (the global decision maker) has the following decision input,

$$I_0 = (u_1, u_2, \ldots, u_n)$$

The Generalized Communication Structure

The representation of decentralized detection systems in terms of the communication structure can describe systems which are connected in the form of a tree and where the decisions flow only in one direction namely towards the fusion center. However, this representation is not adequate for representing decenetralized detection networks with more general network topologies such as the decentralized

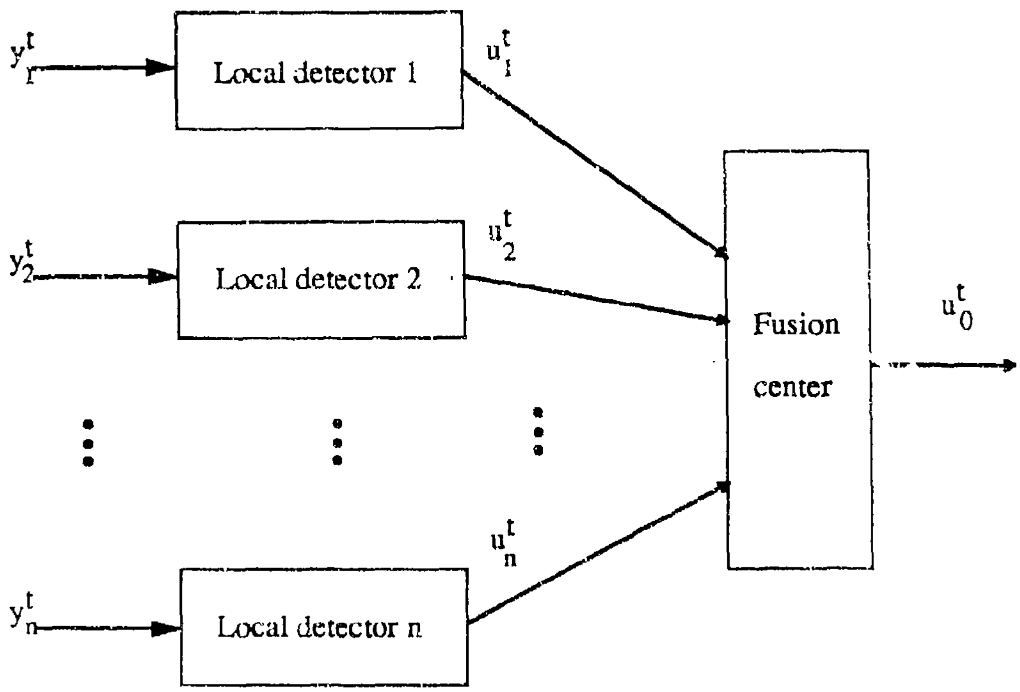

Fig. 4.2: A decentralized detection system

detection network with feedback considered earlier in this dissertation. Therefore, we generalize the definition of the communication structure by including the time parameter t. We assume that each detector in a given system produces a time delay of one unit. Consider the connected graph corresponding to any given decentralized detection network topology where the nodes represent the decision makers and the decisions flow along the directed edges of the graph. Recall the fact that the fusion center is responsible for making the final decision. We organize and label the graph in terms of levels such that the fusion center is at level zero and the level of other nodes is determined by their distances from the fusion center (number of edges traversed from the fusion center to the node under consideration). We illustrate this in Figure 4.3 where a decentralized detection network with a general (non-tree) topology alongwith its corresponding graph is shown. We employ the above connected graph to assign the time index to each of the detectors of the decentralized detection network. The time index of a detector is simply its level in the connected graph. The time indices of the detectors are displayed alongwith the detector number in the communication matrix D. Finally, the input decision vector of the detector corresponding to the k^{th} column is given by:

$$I_k^t = \{u_i^{t+r_i-c_k-1} : D_{ik} = 1;\ for\ all\ i\} \tag{4.2}$$

where

c_k is the time index of the detector corresponding to the k^{th} column.

r_i is the time index of the detector corresponding to the i^{th} row.

For the decentralized detection system of Figure 4.3, the communication matrix is

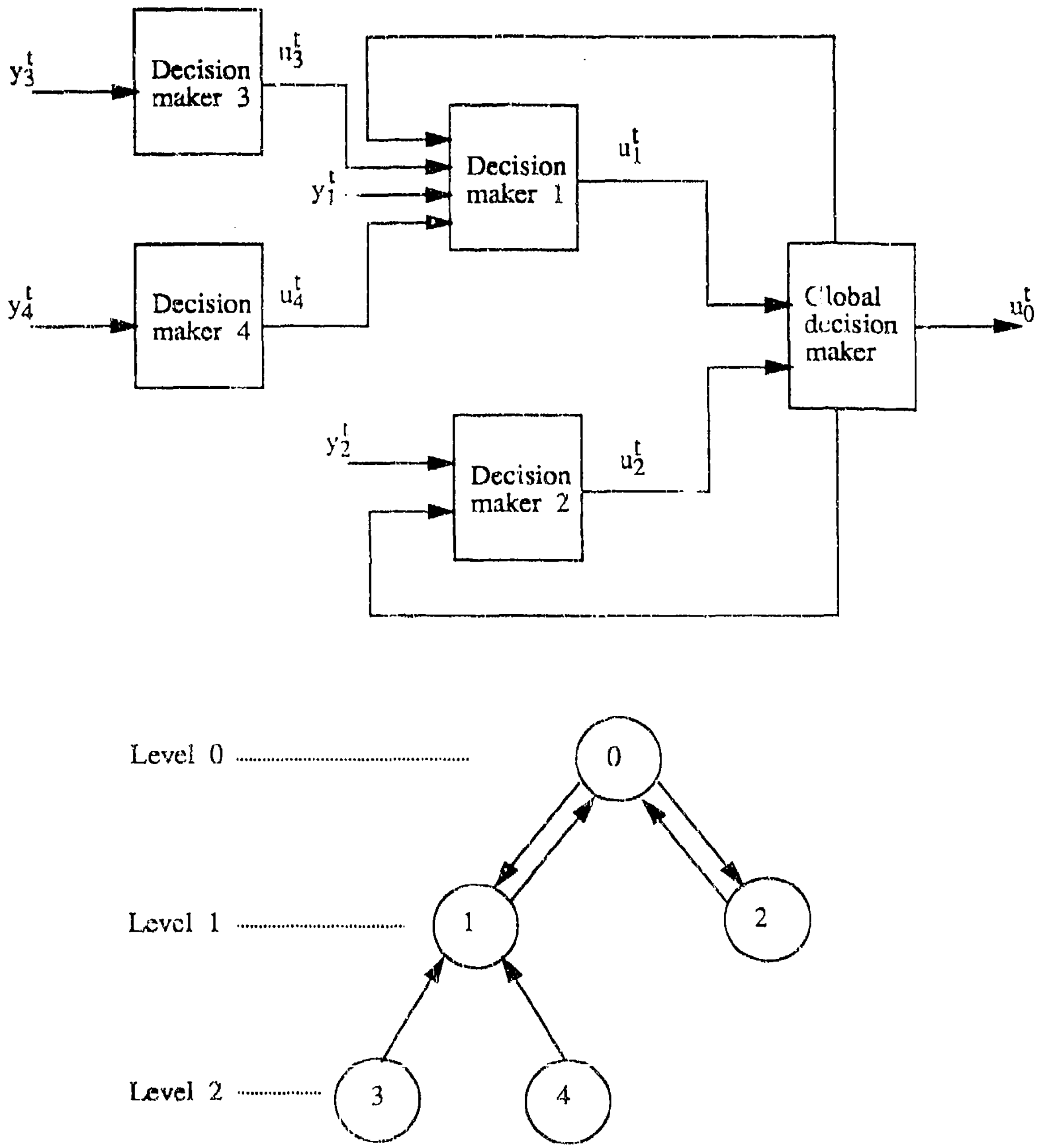

Fig. 4.3: A general (non-tree) decentralized detection system with the corresponding connected graph.

given by,

$$
D = \begin{array}{cc} \textit{time index} & \longrightarrow \\ \downarrow & \textit{det. no.} \\ 1 & 1 \\ 1 & 2 \\ 2 & 3 \\ 2 & 4 \\ 0 & 0 \end{array}
\begin{array}{c} 1\,1\,2\,2\,0 \\ 1\,2\,3\,4\,0 \\ \begin{pmatrix} 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1 \\ 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,1\,0\,0\,0 \end{pmatrix} \end{array}
$$

The decision input vector I_1^t for the first detector is obtained from the first column. The column time index is given by $c_1 = 1$ and

$$I_1^t = \{u_3^{t+2-1-1}, u_4^{t+2-1-1}, u_0^{t+0-1-1}\} = \{u_3^t, u_4^t, u_0^{t-2}\}.$$

Next, we further illustrate the applicability of the generalized representation by considering the following examples.

Example 4.3:

In this example, we look at the serial network of Example 4.1 and obtain the time indices. The communication matrix D is given by

$$
D = \begin{array}{cc} \textit{time index} & \longrightarrow \\ \downarrow & \textit{det. no.} \\ N-1 & 1 \\ N-2 & 2 \\ N-3 & 3 \\ \vdots & \vdots \\ 1 & N-1 \\ 0 & N \end{array}
\begin{array}{c} N-1 \quad \cdots\cdots 0 \\ 1 \;\; 2 \;\; 3 \;\; 4 \;\; \cdots N \\ \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots 0 \\ 0 & 0 & 1 & 0 & \cdots 0 \\ 0 & 0 & 0 & 1 & \cdots 0 \\ \vdots & \vdots & \cdots & \ddots & \ddots 0 \\ 0 & 0 & 0 & 0 & \vdots \;\; 1 \\ 0 & \cdots & \cdots & \cdots & \cdots 0 \end{pmatrix} \end{array}
$$

Once again, the time indices and detector numbers are included for the convenience of the reader. The time index c_j (time index of the detector corresponding to

column k) of the non-zero entries in the matrix D could be written in terms of the time index r_i (time index of the detector corresponding to row i) as follows:

$$c_k = r_i + 1$$

Hence, the input decision vector I_n^t consists of one decision, namely the previous detector decision

$$I_n^t = u_{n-1}^{t+r_i-c_k-1} = u_{n-1}^t$$

This indicates that the decision of the $(n-1)^{th}$ detector is used in the n^{th} detector decision making without any further delay.

Note that the time indices and detector numbers along the columns are repeated along the rows. Therefore, for brevity, from now on we will provide this information only along the rows.

Example 4.4:

We consider the decentralized detection system with a fusion center as given in Example 4.2 and obtain the time indices. The communication matrix D is of dimension (n+1)x(n+1) and is given by:

$$D = \begin{array}{cc} 1 & 1 \\ 1 & 2 \\ \vdots & \vdots \\ 1 & n \\ 0 & 0 \end{array} \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix}$$

It is seen that all the local detectors have the same time index. The time index r_i of the non-zero entries is given by $r_i = 1$, i=1,2,...,n , and the time index c_0 is given by $c_0 = 0$. Hence, the time parameter of the local decisions at the global decision maker is:

$$t + r_i - c_0 - 1 = t + 1 - 0 - 1 = t$$

The decision input at the global decision maker I_0^t is, therefore, given by,

$I_0^t = (u_1^t, u_2^t, \ldots, u_n^t).$

Local detectors have no decision input as seen before.

Example 4.5:

We consider a decentralized detection system with feedback as shown in Figure 4.1. The system consists of n local detectors and a fusion center. The number of levels in this system is the same as that of Example 4.4, hence the same time indices are obtained. The communication matrix D is, therefore, given by:

$$D = \begin{array}{cc} 1 & 1 \\ 1 & 2 \\ \vdots & \vdots \\ 1 & n \\ 0 & 0 \end{array} \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix}$$

Observe the effect of feedback on the matrix D. The bottom row indicates that there is a communication link from the global decision maker (detector 0) to all the local detectors. Note that the decision input of the local detector corresponding to the column k, k=1,2,...,n, has a time index of one, i.e., $c_k = 1$. The decision input of the local detector corresponding to the column k is given by

$$I_k^t = u_0^{t+r_0-c_k-1} = u_0^{t+0-1-1} = u_0^{t-2} \; for\ any\ local\ detector\ k.$$

As seen above, the global decision input to the local detectors has a time parameter of t-2 which indicates that two time delays are encountered, the local detector delay and the global decision maker delay. It is important to note that our earlier results from Chapters 2 and 3 assume that the global decision maker does not account for any time delay. Hence, the time parameter of t-1 was used for the previous global decision in Chapters 2 and 3.

The decision input of the global decision maker is obtained using the 0^{th} column,

$$I_0^t = (u_1^t, u_2^t, ..., u_n^t)$$

The time parameter of the local decisions indicate that all local decisions are used without any time delay.

With our generalized definition of the communication structure, any decentralizd detection system can be represented by a communication matrix. In the next section we derive the decision rules of all the detectors in a decentralized detection system with any configuration represented in terms of its communication matrix.

4.3 The General Decentralized Detection System

4.3.1 System Description and Problem Statement

We consider the binary hypothesis testing problem for a decentralized detection system with any arbitrary configuration (Serial Network, Parallel Network, System with Feedback, etc.). Let the number of detectors in the system be n+1. The block diagram of any detector, say the k^{th} detector, of a decentralized detection system is shown in Figure 4.4. Due to the effect of event sequencing, we associate a time step parameter t with all the system variables. The k^{th} detector of the general system operates as follows: At time step t, the k^{th} detector based on the observation input y_k^t and the decision input I_k^t produces the decision u_k^t using the decision rule $\gamma_k^t(.)$ as follows:

$$u_k^t = \gamma_k^t(y_k^t, I_k^t)$$

We assume that the joint conditional probability density of the observations $p(y_0^t, y_1^t, ..., y_n^t | H_j)$; j = 0, 1; t = 1, 2, ..., T is known a priori. The problem is to find the PBPO solution for the decision rule $\gamma_k^t(.)$, k=0,1, ...,n; t=1, 2, ..., T, so

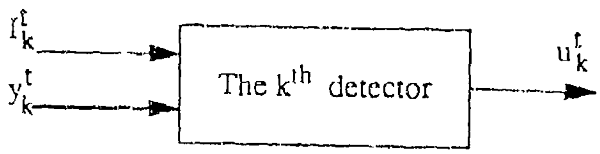

Figure 4.4: Block diagram of the k^{th} detector in a given system.

as to minimize the cost function $J(\Gamma)$ for the final decision u_0^T. We consider the Bayesian formulation where the cost function $J(\Gamma)$ is given by,

$$\begin{aligned} J(\Gamma) = \quad & C_{00}p(u_0^T = 0, H_0) + C_{01}p(u_0^T = 0, H_1) \\ & + C_{10}p(u_0^T = 1, H_0) + C_{11}p(u_0^T = 1, H_1) \end{aligned} \tag{4.3}$$

where C_{ij}; i, j = 0, 1, is the cost of deciding $u_0^T = H_i$ when the true hypothesis is H_j.

The costs C_{ij}; i, j = 0, 1, and the a priori probabilities $p(H_0)$ and $p(H_1)$ are assumed to be known. We rewrite Equation (4.3) in terms of the system probability of false alarm at time step T, p_f^T, and the system probability of detection at time step T, p_d^T, as follows

$$\begin{aligned} J(\Gamma) \quad & = C_f p(u_0^T = 1|H_0) - C_d p(u_0^T = 1|H_1) + C \\ & = C_f p_{f_0}^T - C_d p_{d_0}^T + C \end{aligned} \tag{4.4}$$

where

$$C_f = p(H_0)(C_{10} - C_{00})$$

$$C_d = p(H_1)(C_{01} - C_{11})$$

$$C = p(H_0)C_{00} + p(H_1)C_{01}$$

It should be noted that the cost function $J(\Gamma)$ of Equation (4.4) is independent of the system structure (configuration). Hence, the development up to this point is for a general system. In the next subsection, we derive the PBPO decision rule $\gamma_k^t(.)$ of the k^{th} detector for a general system.

4.3.2 System Optimization

Before proceeding with the system optimization, we make certain simplifying assumptions. We assume that the observations of the general system are spatially as

well as temporally independent. Hence, the a priori knowledge of the conditional probability density functions $p(y_0^t, y_1^t, ..., y_n^t|H_j)$; j = 0, 1; t = 1, 2, ..., T reduces to the a priori knowledge of the individual detector conditional probability densities $p(y_k^t|H_j)$; j=0,1; t=1,2,...,T; k=0,1,...,n. Next, we proceed with the minimization of the cost function given in Equation (4.4). We derive the decision rule for the k^{th} detector shown in Figure 4.4. The result is presented in Theorem 4.1.

THEOREM 4.1

For the binary hypothesis testing problem in a general decentralized detection system, the PBPO decision rule of the k^{th} detector (Figure 4.4) that minimizes the Bayesian cost function associated with the global decision at the final time T is given by:

$$\gamma_k^t(y_k^t, I_k^t) = u_k^t = \begin{cases} 1 & if\ \Lambda(y_k^t) > \eta_k^t(I_k^t) \\ 0 & otherwise \end{cases} \tag{4.5}$$

for all k=0,1,...,n; t=1,2,...,T; where $\eta_k^t(I_k^t)$ is the threshold of the k^{th} detector at time step t defined as:

$$\eta_k^t(I_k^t) = \frac{C_f g^t(T,0) f^t(u_k^t, 0) p(I_k^t|H_0)}{C_d g^t(T,1) f^t(u_k^t, 1) p(I_k^t|H_1)} \tag{4.6}$$

and

$$g^t(T,i) = p(u_0^T = 1|u_0^t = 1, H_i) - p(u_0^T = 1|u_0^t = 0, H_i)$$

$$f^t(u_k^t, i) = p(u_0^t = 1|u_k^t = 1, H_i) - p(u_0^t = 1|u_k^t = 0, H_i)$$

Proof:

We start with Equation (4.4) and expand it in terms of the k^{th} decision at time step t, u_k^t, the decision input I_k^t, the observation y_k^t, and the global decision at time step t, u_0^t as follows:

$$J(T) = \sum_{I_k^t, u_0^t, u_k^t} \int_{y_k^t} C_f p(u_0^T = 1, I_k^t, u_0^t, u_k^t, y_k^t|H_0)$$

$$-C_d p(u_0^T = 1, I_k^t, u_0^t, u_k^t, y_k^t | H_1) + C \tag{4.7}$$

Conditioning on u_0^t, I_k^t, u_k^t, and y_k^t, Equation (4.7) is rewritten as:

$$\begin{aligned} J(\Gamma) = \sum_{I_k^t, u_0^t, u_k^t} \quad & \int_{y_k^t} C_f p(u_0^T = 1 | u_0^t, u_k^t, I_k^t, y_k^t, H_0) \\ & \times p(u_0^t, u_k^t, y_k^t, I_k^t | H_0) \\ & -C_d p(u_0^T = 1 | u_0^t, u_k^t, I_k^t, y_k^t, H_1) \\ & \times p(u_0^t, u_k^t, I_k^t, y_k^t | H_1) + C \end{aligned} \tag{4.8}$$

Writing the cost function J(Γ) of (4.8) explicitly in terms of all the possibilities of the global decision u_0^t and conditioning further on u_k^t, I_k^t, and y_k^t, we have

$$\begin{aligned} J(\Gamma) = \sum_{I_k^t, u_k^t} \quad & \int_{y_k^t} C_f p(u_0^T = 1 | u_0^t = 1, u_k^t, I_k^t, y_k^t, H_0) \\ & \times p(u_0^t = 1 | u_k^t, y_k^t, I_k^t, H_0) p(u_k^t, y_k^t, I_k^t | H_0) \\ & -C_d p(u_0^T = 1 | u_0^t = 1, u_k^t, I_k^t, y_k^t, H_1) \\ & \times p(u_0^t = 1 | u_k^t, I_k^t, y_k^t, H_1) p(u_k^t, I_k^t, y_k^t | H_1) \\ & +C_f p(u_0^T = 1 | u_0^t = 0, u_k^t, I_k^t, y_k^t, H_0) \\ & \times p(u_0^t = 0 | u_k^t, I_k^t, y_k^t, H_0) p(u_k^t, I_k^t, y_k^t | H_0) \\ & -C_d p(u_0^T = 1 | u_0^t = 0, u_k^t, I_k^t, y_k^t, H_1) \\ & \times p(u_0^t = 0 | u_k^t, I_k^t, y_k^t, H_1) p(u_k^t, I_k^t, y_k^t | H_1) + C \end{aligned} \tag{4.9}$$

We observe that the final global decision $u_0^T = 1$ given the global decision at time t, $u_0^t = j$ and the hypothesis H_i does not depend on u_k^t, I_k^t, and y_k^t. Hence, we rewrite (4.9) by factoring out the common terms and substituting $p(u_0^t = 0|.)$ by $1 - p(u_0^t = 1|.)$

$$\begin{aligned} J(\Gamma) = \sum_{I_k^t, u_k^t} \quad & \int_{y_k^t} C_f p(u_k^t, y_k^t, I_k^t | H_0) [p(u_0^T = 1 | u_0^t = 1, H_0) \\ & \times p(u_0^t = 1 | u_k^t, y_k^t, I_k^t, H_0) + p(u_0^T = 1 | u_0^t = 0, H_0) \\ & \times (1 - p(u_0^t = 1 | u_k^t, I_k^t, y_k^t, H_0))] \\ & -C_d p(u_k^t, y_k^t, I_k^t | H_1) [p(u_0^T = 1 | u_0^t = 1, H_1) \end{aligned}$$

$$\times p(u_0^t = 1|u_k^t, y_k^t, I_k^t, H_1) + p(u_0^T = 1|u_0^t = 0, H_1)$$
$$\times(1 - p(u_0^t = 1|u_k^t, I_k^t, y_k^t, H_1))] + C \quad (4.10)$$

Multiplying out the term $(1 - p(u_0^t = 1|.))$ and rearranging, we have

$$J(\Gamma) = \sum_{I_k^t, u_k^t} \int_{y_k^t} C_f p(u_k^t, y_k^t, I_k^t|H_0) p(u_0^t = 1|u_k^t, y_k^t, I_k^t, H_0)$$
$$\times[p(u_0^T = 1|u_0^t = 1, H_0) - p(u_0^T = 1|u_0^t = 0, H_0)]$$
$$+C_f p(u_k^t, y_k^t, I_k^t|H_0) p(u_0^T = 1|u_0^t = 0, H_0)$$
$$-C_d p(u_k^t, y_k^t, I_k^t|H_1) p(u_0^t = 1|u_k^t, y_k^t, I_k^t, H_1)$$
$$\times[p(u_0^T = 1|u_0^t = 1, H_1) - p(u_0^T = 1|u_0^t = 0, H_1)]$$
$$-C_d p(u_k^t, y_k^t, I_k^t|H_1) p(u_0^T = 1|u_0^t = 0, H_1) + C \quad (4.11)$$

Letting $p(u_0^T = 1|u_0^t = 1, H_i) - p(u_0^T = 1|u_0^t = 0, H_i) = g^t(T, i)$ and conditioning (4.11) further on y_k^t and I_k^t, we have

$$J(\Gamma) = \sum_{I_k^t, u_k^t} \int_{y_k^t} C_f p(u_k^t|y_k^t, I_k^t, H_0) p(y_k^t, I_k^t|H_0)$$
$$\times p(u_0^t = 1|u_k^t, y_k^t, I_k^t, H_0) g^t(T, 0)$$
$$+C_f p(u_k^t|y_k^t, I_k^t, H_0) p(y_k^t, I_k^t|H_0) p(u_0^T = 1|u_0^t = 0, H_0)$$
$$-C_d p(u_k^t|y_k^t, I_k^t, H_1) p(y_k^t, I_k^t|H_1)$$
$$\times p(u_0^t = 1|u_k^t, y_k^t, I_k^t, H_1) g^t(T, 1)$$
$$-C_d p(u_k^t|y_k^t, I_k^t, H_1) p(y_k^t, I_k^t|H_1) p(u_0^T = 1|u_0^t = 0, H_1) + C \quad (4.12)$$

We note that the k^{th} detector decision u_k^t given the observation y_k^t and the decision input I_k^t does not depend on the hypothesis present. Next, we rewrite the cost function $J(\Gamma)$ of (4.12) in terms of all possibilities of the decision u_k^t, hence

$$J(\Gamma) = \sum_{I_k^t} \int_{y_k^t} p(u_k^t = 1|y_k^t, I_k^t)[C_f p(y_k^t, I_k^t|H_0)$$
$$\times\{p(u_0^t = 1|u_k^t = 1, y_k^t, I_k^t, H_0) g^t(T, 0) + p(u_0^T = 1|u_0^t = 0, H_0)\}$$
$$-C_d p(y_k^t, I_k^t|H_1) \times \{p(u_0^t = 1|u_k^t = 1, y_k^t, I_k^t, H_1) g^t(T, 1)$$

$$+p(u_0^T = 1|u_0^t = 0, H_1)\}] + p(u_k^t = 0|y_k^t, I_k^t)[C_f p(y_k^t, I_k^t|H_0)$$
$$\times\{p(u_0^t = 1|u_k^t = 0, y_k^t, I_k^t, H_0)g^t(T,0) + p(u_0^T = 1|u_0^t = 0, H_0)\}$$
$$-C_d p(y_k^t, I_k^t|H_1) \times \{p(u_0^t = 1|u_k^t = 0, y_k^t, I_k^t, H_1)g^t(T,1)$$
$$+p(u_0^T = 1|u_0^t = 0, H_1)\}] + C \tag{4.13}$$

We observe that the global decision at time t, u_0^t given $u_k^t = j$ and the hypothesis H_i does not depend on y_k^t and I_k^t. In addition, substituting $p(u_k^t = 0|y_k^t, I_k^t)$ by $1 - p(u_k^t = 1|y_k^t, I_k^t)$ in (4.13) and rearranging, we have

$$J(\Gamma) = \sum_{I_k^t} \int_{y_k^t} p(u_k^t = 1|y_k^t, I_k^t)[C_f p(y_k^t, I_k^t|H_0)$$
$$\times\{p(u_0^t = 1|u_k^t = 1, H_0)g^t(T,0) + p(u_0^T = 1|u_0^t = 0, H_0)\}$$
$$-C_d p(y_k^t, I_k^t|H_1)\{p(u_0^t = 1|u_k^t = 1, H_1)g^t(T,1)$$
$$+p(u_0^T = 1|u_0^t = 0, H_1)\} - C_f p(y_k^t, I_k^t|H_0)$$
$$\times\{p(u_0^t = 1|u_k^t = 0, H_0)g^t(T,0) + p(u_0^T = 1|u_0^t = 0, H_0)\}$$
$$+C_d p(y_k^t, I_k^t|H_1)\{p(u_0^t = 1|u_k^t = 0, H_1)g^t(T,1)$$
$$+p(u_0^T = 1|u_0^t = 0, H_1)\}] + C_f p(y_k^t, I_k^t|H_0)$$
$$\times\{p(u_0^t = 1|u_k^t = 0, H_0)g^t(T,0) + p(u_0^T = 1|u_0^t = 0, H_0)\}$$
$$-C_d p(y_k^t, I_k^t|H_1) \times \{p(u_0^t = 1|u_k^t = 0, H_1)g^t(T,1)$$
$$+p(u_0^T = 1|u_0^t = 0, H_1)\} + C \tag{4.14}$$

We observe that the last three terms of Equation (4.14) are not involved in the optimization of the k^{th} detector. We discard these terms in the subsequent equations and denote the new cost function by $J^1(\Gamma)$. Rearranging by further factorization of common terms in Equation (4.14), we get

$$J^1(\Gamma) = \sum_{I_k^t} \int_{y_k^t} p(u_k^t = 1|y_k^t, I_k^t)[C_f p(y_k^t, I_k^t|H_0)$$
$$\times\{p(u_0^t = 1|u_k^t = 1, H_0)g^t(T,0) + p(u_0^T = 1|u_0^t = 0, H_0)$$
$$-p(u_0^t = 1|u_k^t = 0, H_0)g^t(T,0) - p(u_0^T = 1|u_0^t = 0, H_0)\}$$
$$-C_d p(y_k^t, I_k^t|H_1)\{p(u_0^t = 1|u_k^t = 1, H_1)g^t(T,1)$$

$$+p(u_0^T = 1|u_0^t = 0, H_1) - p(u_0^t = 1|u_k^t = 0, H_1)g^t(T,1)$$
$$--p(u_0^T = 1|u_0^t = 0, H_1)\}] \quad (4.15)$$

Canceling out the equal terms and rearranging by further factorization of common terms, Equation (4.15) is rewritten as:

$$J^1(\Gamma) = \sum_{I_k^t} \int_{y_k^t} p(u_k^t = 1|y_k^t, I_k^t)[C_f p(y_k^t, I_k^t|H_0)g^t(T,0)$$
$$\times\{p(u_0^t = 1|u_k^t = 1, H_0) - p(u_0^t = 1|u_k^t = 0, H_0)\}$$
$$--C_d p(y_k^t, I_k^t|H_1)g^t(T,1)$$
$$\times\{p(u_0^t = 1|u_k^t = 1, H_1) - p(u_0^t = 1|u_k^t = 0, H_1)\}] \quad (4.16)$$

Letting $p(u_0^t = 1|u_k^t = 1, H_i) - p(u_0^t = 1|u_k^t = 0, H_i) = f^t(u_k^t, i)$, we rewrite (4.16) as:

$$J^1(\Gamma) = \sum_{I_k^t} \int_{y_k^t} p(u_k^t = 1|y_k^t, I_k^t)$$
$$\times[C_f p(y_k^t, I_k^t|H_0)g^t(T,0)f^t(u_k^t,0)$$
$$--C_d p(y_k^t, I_k^t|H_1)g^t(T,1)f^t(u_k^t,1)] \quad (4.17)$$

The cost function $J^1(\Gamma)$ of (4.17) is minimized if we choose

$$p(u_k^t = 1|y_k^t, I_k^t) = \begin{array}{ll} 1 & if\ A_1 > A_0 \\ 0 & otherwise \end{array} \quad (4.18)$$

where

$$A_1 = C_d p(y_k^t, I_k^t|H_1)g^t(T,1)f^t(u_k^t,1)$$
$$A_0 = C_f p(y_k^t, I_k^t|H_0)g^t(T,0)f^t(u_k^t,0)$$

The k^{th} detector decision rule $\gamma_k^t(.)$ of the general system is given by rewriting (4.18) as:

$$\gamma_k^t(y_k^t, I_k^t) = u_k^t = \begin{array}{ll} 1 & if\ \Lambda(y_k^t, I_k^t) > \mu_k^t \\ 0 & otherwise \end{array} \quad (4.19)$$

where μ_k^t is the threshold of the k^{th} detector at time step t defined as

$$\mu_k^t = \frac{C_f g^t(T,0) f^t(u_k^t,0)}{C_d g^t(T,1) f^t(u_k^t,1)}$$

Using the assumption of temporal and spatial independence of observations in the general system, the k^{th} detector observation y_k^t is independent of the k^{th} detector decision input I_k^t. Hence, the likelihood ratio is separable as follows:

$$\Lambda(y_k^t, I_k^t) = \Lambda(y_k^t) \times \Lambda(I_k^t)$$

Substituting this result in Equation (4.19) and rearranging, we have

$$\gamma_k^t(y_k^t, I_k^t) = u_k^t = \begin{cases} 1 & if\ \Lambda(y_k^t) > \eta_k^t(I_k^t) \\ 0 & otherwise \end{cases} \tag{4.20}$$

where $\eta_k^t(I_k^t)$ is a multivalued threshold of the k^{th} detector at time step t defined as:

$$\eta_k^t(I_k^t) = \frac{C_f g^t(T,0) f^t(u_k^t,0) p(I_k^t|H_0)}{C_d g^t(T,1) f^t(u_k^t,1) p(I_k^t|H_1)} \tag{4.21}$$

as given in Equation (4.6).

Q.E.D.

It should be noted that Equation (4.19) represents the general decision rule for any detector of a decentralized detection system with an arbitrary configuration. Moreover, the general decision rule of any detector k at any time step t is based on the likelihood ratio of the input to that detector. Thus, the decision rule at the global decision maker can be obtained from the above general result. The result is given next.

Lemma 4.1:

For a general decentralized detection system the PBPO decision rule $\gamma_0^T(.)$ of the global decision maker that minimizes the Bayesian cost function for the binary

hypothesis testing problem is given by

$$\gamma_0^T(I_0^T, y_0^T) = u_0^t = \begin{cases} 1 & if\ \Lambda(I_0^T, y_0^T) > \dfrac{C_f}{C_d} \\ 0 & otherwise \end{cases} \tag{4.22}$$

where I_0^T is the decision input of the global decision maker and y_0^t is the input observation of the global decsision maker (if any).

Proof:

The global decision rule of (4.22) results directly from the general decision rule (4.19) by letting k = 0, t = T, and observing the following:

$$\begin{aligned} g^T(T,i) = & \quad p(u_0^T = 1 | u_0^T = 1, H_i) - p(u_0^T = 1 | u_0^T = 0, H_i) \\ = & \quad 1 - 0 = 1 \end{aligned}$$

and for t=1,2,...,T

$$\begin{aligned} f^t(u_0^t, i) = & \quad p(u_0^t = 1 | u_0^t = 1, H_i) - p(u_0^t = 1 | u_0^t = 0, H_i) \\ = & \quad 1. \end{aligned}$$

Hence, the threshold of (4.19) reduces to:

$$\eta_k^T = \frac{C_f}{C_d}$$

resulting in Equation (4.22).

Q.E.D.

The decision rule of (4.22) is a general global decision rule in that the global decision maker may also make direct observations of the phenomenon in addition to the decisions received from the other detectors. The observation term y_0^t is to be dropped if there is no direct observation at the global decision maker. The result of Lemma 4.1 agrees with the global decision rule at time step T of the FSS

problem given in Theorem 2.3 if $I_0^T = U^T$ is used. The decision rules at the local detectors for the FSS problem considered in Chapter 2 can also be obtained. It is demonstrated in Lemma 4.2.

Lemma 4.2

For the decentralized detection system with feedback, the decision rule at the k^{th} detector for the FSS problem is given by Theorem 4.1 with $I_k^t = u_0^{t-1}$, i.e.,

$$\gamma_k^t(y_k^t, u_0^{t-1}) = u_k^t = \begin{cases} 1 & if\ \Lambda(y_k^t) > \eta_k^t(u_0^{t-1}) \\ 0 & otherwise \end{cases} \tag{4.23}$$

for k=1,2,...,n and t=1,2,...,T; where

$$\eta_k^t(u_0^{t-1}) = \frac{C_f g^T(t,0) f^t(u_k^t,0) p(u_0^{t-1}|H_0)}{C_d g^T(t,1) f^t(u_k^t,1) p(u_0^{t-1}|H_1)} \tag{4.24}$$

Proof:

It is seen that $g^T(t,0)$ is the same as defined in Theorem 2.4. Expanding $f^t(u_k^t, i)$, i = 0, 1, in terms of the decision vector U_k^t, we have

$$f^t(u_k^t, i) = p(u_0^t = 1|u_k^t = 1, H_i) - p(u_0^t = 1|u_k^t = 0, H_i)$$

$$f^t(u_k^t, i) = \sum_{U_k^t} p(u_0^t = 1, U_k^t|u_k^t = 1, H_i) - p(u_0^t = 1, U_k^t|u_k^t = 0, H_i)$$

Conditioning on U_k^t and rearranging

$$f^t(u_k^t, i) = \sum_{U_k^t} [p(u_0^t = 1|u_k^t = 1, U_k^t, H_i) - p(u_0^t = 1|u_k^t = 0, U_k^t, H_i)] \times p(U_k^t|H_i) \tag{4.25}$$

Observe that the global decision u_0^t given all of the local decisions, i.e., u_k^t and U_k^t, does not depend on the hypothesis present. Hence

$$f^t(u_k^t, i) = \sum_{U_k^t} [p(u_0^t = 1|u_k^t = 1, U_k^t) - p(u_0^t = 1|u_k^t = 0, U_k^t)] p(U_k^t|H_i)$$

Note that $[p(u_0^t = 1|u_k^t = 1, U_k^t) - p(u_0^t = 1|u_k^t = 0, U_k^t)] = f(U_k^t)$ as defined in Equation (2.45). Thus,

$$f^t(u_k^t, i) = \sum_{U_k^t} f(U_k^t) p(U_k^t | H_i) \tag{4.26}$$

Substituting the above result for i = 0, 1 in Equation (4.24) and recalling the spatial and temporal independence of observations yields Equation (2.46) of Theorem 2.4.

Q.E.D.

Next, we turn to the results in Section 2.3 where the FBPO solution was obtained without the knowledge of the final time T. In other words, the system was optimized with the assumption that the decision process could end at any time t. In Lemma 4.3, we obtain this result using our unified approach.

Lemma 4.3:

For the decentralized detection system with feedback but without memory, the decision rule at the k^{th} detector is obtained by letting $I_k^t = u_0^{t-1}$ and time step T = t in Theorem 4.1, i.e.,

$$\gamma_k^t(y_k^t, u_0^{t-1}) = u_k^t = \begin{cases} 1 & if\ \Lambda(y_k^t) > \eta_k^t(u_0^{t-1}) \\ 0 & otherwise \end{cases} \tag{4.27}$$

for k=1,2,...,n; where

$$\eta_k^t(u_0^{t-1}) = \frac{C_f f^t(u_k^t, 0) p(u_0^{t-1} | H_0)}{C_d f^t(u_k^t, 1) p(u_0^{t-1} | H_1)} \tag{4.28}$$

Proof:

As explained in Example 4.5, the decision input to the k^{th} detector I_k^t is given by $I_k^t = u_0^{t-1}$. The result of substituting $I_k^t = u_0^{t-1}$ in equations (4.5) and (4.6) of Theorem 4.1 is straightforward. The result of substituting T = t needs to be examined. We look at the function $g^t(T, i)$ which by letting T = t, we have

$$g^t(t, i) = p(u_0^t = 1 | u_0^t = 1, H_i) - p(u_0^t = 1 | u_0^t = 0, H_i)$$

Using the facts that

$p(u_0^t = 1 | u_0^t = 1, H_i) = 1$

$p(u_0^t = 1 | u_0^t = 0, H_i) = 0$

The function $g^t(T = t, i) = 1$.

Substituting this result for i = 0, 1 in Equation (4.6) yields Equation (4.28). Following similar steps as in Lemma 4.2 for the development of $f^t(u_k^t, i)$, the threshold given in Equation (4.28) is the same as that of Equation (2.6) of Theorem 2.2.

Q.E.D.

The local and global decision rules of Chapter 3 can be verified in a similar fashion. The communication structure of the decentralized detection system with feedback and memory is the same as that of the system without memory. Hence, the decision rule design is the same in both systems. At this stage we turn to the results in the literature where we look at the general formulation attempted by Reibman and Nolte [9] and show that their results are a special case of our results. It should be noted that in the literature, the same detector at two different time instants is considered as two different detectors. Hence, the time parameter t does not need to be taken into account here. The result is presented in Lemma 4.4 next.

Lemma 4.4:

For the binary hypothesis testing problem in a decentralized detection system, the PBPO decision rule at the k^{th} detector (Figure 4.5) that minimizes the Bayesian cost function of the final global decision is given by:

$$\gamma_k(y_k, I_k) = \begin{cases} 1 & if\ \Lambda(y_k) > \eta_k(I_k) \\ 0 & otherwise \end{cases} \tag{4.29}$$

where I_k is the decision input of the k^{th} detector and $\eta(I_k)$ is the threshold of the

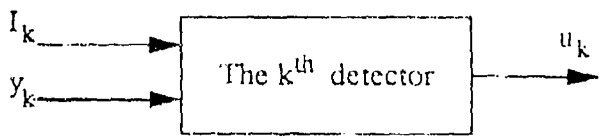

Figure 4.5: Block diagram of the k^{th} detector in a given system.

k^{th} detector defined as:

$$\eta(I_k) = \frac{C_f f(u_k, 0) p(I_k|H_0)}{C_d f(u_k, 1) p(I_k|H_1)} \tag{4.30}$$

and

$$f(u_k, i) = p(u_0 = 1|u_k = 1, H_i) - p(u_0 = 1|u_k = 0, H_i)$$

Proof:

The above results are obtained simply by dropping the superscript t in the results of Theorem 4.1. Since in this formulation, a detector operating at two different time instants is considered as two different detectors. The term $g^t(T, i) = 1$ because the global decision maker operates only once (t=T). It should be noted that the results of Lemma 4.4 agree with that of Reibman and Nolte [9] with the $f(u_k, j)$ term in Equation (4.30) expanded over all possibilities of the local decisions that are input of the global decision maker.

Q.E.D.

In Lemma 4.5 next, we verify the results of the serial system using Lemma 4.4.

Lemma 4.5:

For a serial system consisting of N detectors, the k^{th} detector decision rule is given by Lemma 4.4 with $I_k = u_{k-1}$, namely

$$\gamma_k(y_k, u_{k-1}) = \begin{cases} 1 & if\ \Lambda(y_k) > \eta_k(u_{k-1}) \\ 0 & otherwise \end{cases} \tag{4.31}$$

where $\eta(u_{k-1})$ is the threshold of the k^{th} detector defined as:

$$\eta(u_{k-1}) = \frac{C_f f(u_k, 0) p(u_{k-1}|H_0)}{C_d f(u_k, 1) p(u_{k-1}|H_1)} \tag{4.32}$$

and

$$f(u_k, i) = p(u_0 = 1|u_k = 1, H_i) - p(u_0 = 1|u_k = 0, H_i)$$

Proof:

The above results are obtained by a straightforward substitution of $I_k = u_{k-1}$ in Lemma 4.4. It should be observed that the 0^{th} detector corresponds to the N^{th} detector in the serial system. Furthermore, for k=1, there is no decision input I_k, and for k=0, the term $f(u_k, j) = 1$ resulting in the decision rule at the final detector.

Q.E.D.

In order to demonstrate the versatility of our approach, we apply it to a more complex decentralized detection configuration next.

4.3.3 Decentralized Detection with Peer Communication

We consider the binary hypothesis testing problem for the decentralized detection system with peer communication shown in Figure 4.6. In this system, the k^{th} local detector communicates its decision to the global decision maker as well as all other local detectors. The system operates as follows: At time step t, the k^{th} local detector makes the local decision u_k^t based on its current observation y_k^t, its previous observations $Y_{t-1,k}$, and other detector decisions $u_1^{t-1}, \ldots, u_{k-1}^{t-1}, u_{k+1}^{t-1}, \ldots, u_n^{t-1}$ that are transmitted to it. Let the number of local detectors be n. The number of levels for this system is the same as that of the conventional decentralized detection system of Example 4.4, hence the same time indices are obtained. The

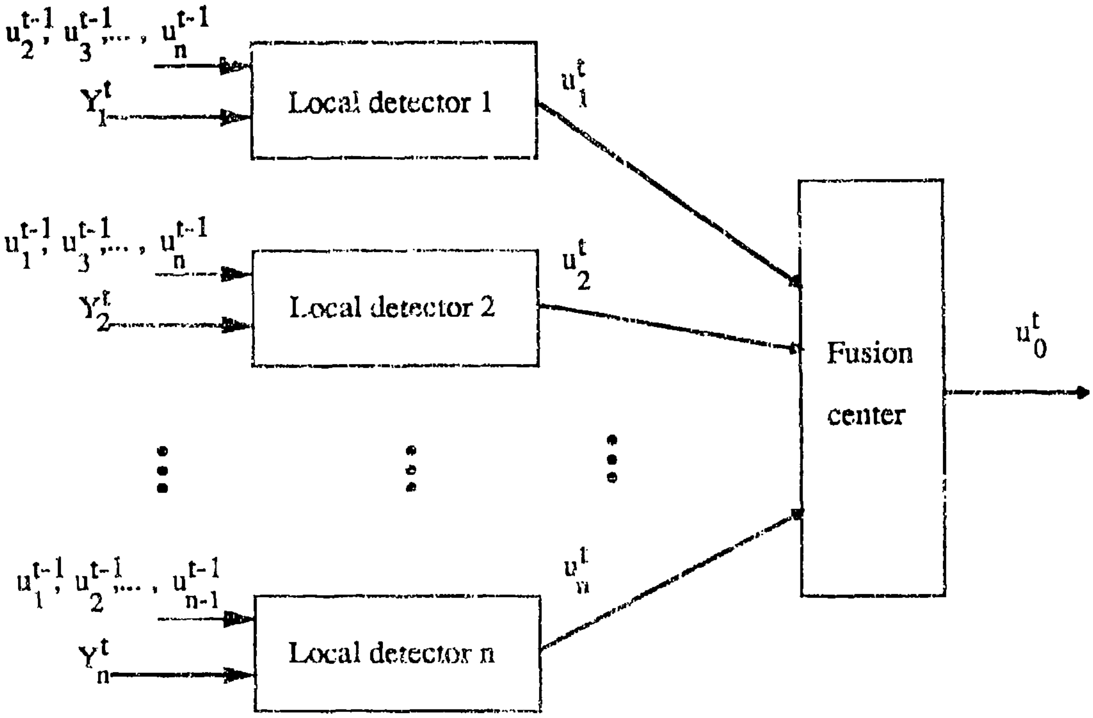

Fig. 4.6: A decentralized detection system with peer communication

communication matrix D is, therefore, given by

$$D = \begin{array}{cc} 1\,1 \\ 1\,2 \\ 1\,3 \\ \vdots\ \vdots \\ 1\,n \\ 2\,0 \end{array} \begin{pmatrix} 0 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 0 \end{pmatrix}$$

Observe that the 0^{th} detector (the global decision maker) does not transmit its decision to the other detectors (row elements are all 0). The decision input of the detector corresponding to column k is therefore given by:

$$I_k^t = u_1^{t-1}, \ldots, u_{k-1}^{t-1}, u_{k+1}^{t-1}, \ldots, u_n^{t-1} \; \textit{for any local detector } k.$$

The decision rule of the k^{th} detector is presented in Lemma 4.6 next.

Lemma 4.6

For the binary hypothesis testing problem of a decentralized detection system with peer communication shown in Figure 4.6, the PBPO decision rule of the k^{th} local detector that minimizes the Bayesian cost function is given by Theorem 4.1 with T=t, namely

$$\gamma_k^t(Y_{t,k}, I_k^t) = u_k^t = \begin{cases} 1 & if\ \Lambda(Y_{t,k}) > \eta_k^t(I_k^t) \\ 0 & otherwise \end{cases} \tag{4.33}$$

where $\eta_k^t(I_k^t)$ is the threshold at time step t defined as

$$\eta_k^t(I_k^t) = \frac{C_f f(u_k^t, 0) p(I_k^t|H_0)}{C_d f(u_k^t, 1) p(I_k^t|H_1)} \tag{4.34}$$

$$f(u_k^t, i) = p(u_0^t = 1|u_k^t = 1, H_i) - p(u_0^t = 1|u_k^t = 0, H_i)$$

$$I_k^t = u_1^{t-1}, \ldots, u_{k-1}^{t-1}, u_{k+1}^{t-1}, \ldots, u_n^{t-1} \; \textit{for any local detector } k$$

Proof:

Straightforward substitution of $I_k^t = u_1^{t-1}, \ldots, u_{k-1}^{t-1}, u_{k+1}^{t-1}, \ldots, u_n^{t-1}$; k=1,2,...,n, and T=t in Theorem 4.1 yields the desired decision rule of Lemma 4.6. The decision rule of the global decision maker is the same as that of the conventional decentralized detection system since the available information at the global decision maker are the same.

Q.E.D.

Next, we present a numerical example utilizing the results obtained for the decentralized detection system with peer communication and compare the performance to the conventional decentralized detection system and the decentralized detection system with feedback and memory.

Example 4.6:

We pursue the example of Chapter 3. Briefly, the system consists of two local detectors and a fusion center. The OR fusion rule is used. The input observations are assumed to have a Rayleigh distribution. The probability of error p_e^t for this system is given by

$$p_e^t = p_m^t p(H_1) + p_f^t p(H_0) \tag{4.35}$$

where p_m^t and p_f^t are the system probability of miss and false alarm respectively. Since the fusion rule is the OR rule, the error probabilities can be written in terms of the local detector error probabilities $p_{m_i}^t$ and $p_{f_i}^t$ as follows

$$p_m^t = (p_{m_i}^t)^2 \tag{4.36}$$

and

$$p_f^t = 1 - (1 - p_{f_i}^t)^2 \tag{4.37}$$

For i=1, the local detector probability of miss at time t, $p^t_{m_1}$ is found by conditioning on the second detector decision at time t-1, u_2^{t-1} as follows

$$\begin{aligned} p^t_{m_1} &= p(u_1^t = 0|H_1) \\ &= \sum_{u_2^{t-1}} p(u_1^t = 0|u_2^{t-1}, H_1)p(u_2^{t-1}|H_1) \end{aligned} \tag{4.38}$$

Expanding in terms of all possibilities of u_2^{t-1}, we have

$$\begin{aligned} p^t_{m_1} = \; & p(u_1^t = 0|u_2^{t-1} = 1, H_1)p(u_2^{t-1} = 1|H_1) \\ & + p(u_1^t = 0|u_2^{t-1} = 0, H_1)p(u_2^{t-1} = 0|H_1) \end{aligned} \tag{4.39}$$

Substituting $p(u_2^{t-1} = 1|H_1) = 1 - p(u_2^{t-1} = 0|H_1)$ and rearranging, we have

$$\begin{aligned} p^t_{m_1} = \; & p(u_2^{t-1} = 0|H_1)[p(u_1^t = 0|u_2^{t-1} = 0, H_1) - p(u_1^t = 0|u_2^{t-1} = 1, H_1)] \\ & + p(u_1^t = 0|u_2^{t-1} = 1, H_1) \end{aligned} \tag{4.40}$$

Since local detectors are assumed to have equal SNR, the following holds

$$p(u_2^{t-1} = 0|H_1) = p^{t-1}_{m_2} = p^{t-1}_{m_1} = p^{t-1}_{m_i}$$

Hence, Equation (4.40) is rewritten as

$$\begin{aligned} p^t_{m_i} = \; & p^{t-1}_{m_i}[p^t_{m_i}(u_i^{t-1} = 0) - p^t_{m_i}(u_i^{t-1} = 1)] \\ & + p^t_{m_i}(u_i^{t-1} = 1) \end{aligned} \tag{4.41}$$

Similarly, the probability of false alarm of the local detector is given by

$$\begin{aligned} p^t_{f_i} = \; & p^{t-1}_{f_i}[p^t_{f_i}(u_i^{t-1} = 1) - p^t_{f_i}(u_i^{t-1} = 0)] \\ & + p^t_{f_i}(u_i^{t-1} = 0) \end{aligned} \tag{4.42}$$

Substituting Equations (4.41) and (4.42) in (4.36) and (4.37) and then substituting the results in (4.35) yields the system probability of error.

The probability of error vs. SNR is plotted in Figure 4.7 for various values of the number of samples per detector. Moreover, for the SNR value of 5 dB, we plot

the system probability of error vs. the number of observation samples per detector in Figure 4.8 for the decentralized detection system with peer communication, the conventional decentralized detection system and the decentralized detection system with feedback and memory.

The plot in Figure 4.7 shows that the probability of error for the decentralized detection system with peer communication decreases as the number of samples per detector increases and SNR value increases. The plot in Figure 4.8 shows the probability of error for three decentralized detection systems for the SNR value of 5dB. It is seen that the probability of error of the decentralized detection system with peer communication is less than that of the conventional decentralized detection system. On the other hand, the decentralized detection system with feedback and memory has the least probability of error for a given number of samples. Similar results for the AND fusion rule are obtained. The probability of error vs. SNR is ploted in Figure 4.10. In Figure 4.9, the probability of error, for the SNR value of 5, is ploted for the decentralized detection system with peer communication, the system with feedback, and the conventional decentralized detection system.

It should be noted that for the case of two local detectors, the decentralized detection system with feedback and memory outperforms the decentralized detection system with peer communication. Intuitively however, the decentralized detection system with peer communication should outperform the decentralized detection system with feedback and memory, which is the case when the system has more than two local detecors.

4.4 Discussion

In this chapter, we have presented a unified approach to the study of a decentralized detection system with any configuration. In this approach, we represent the

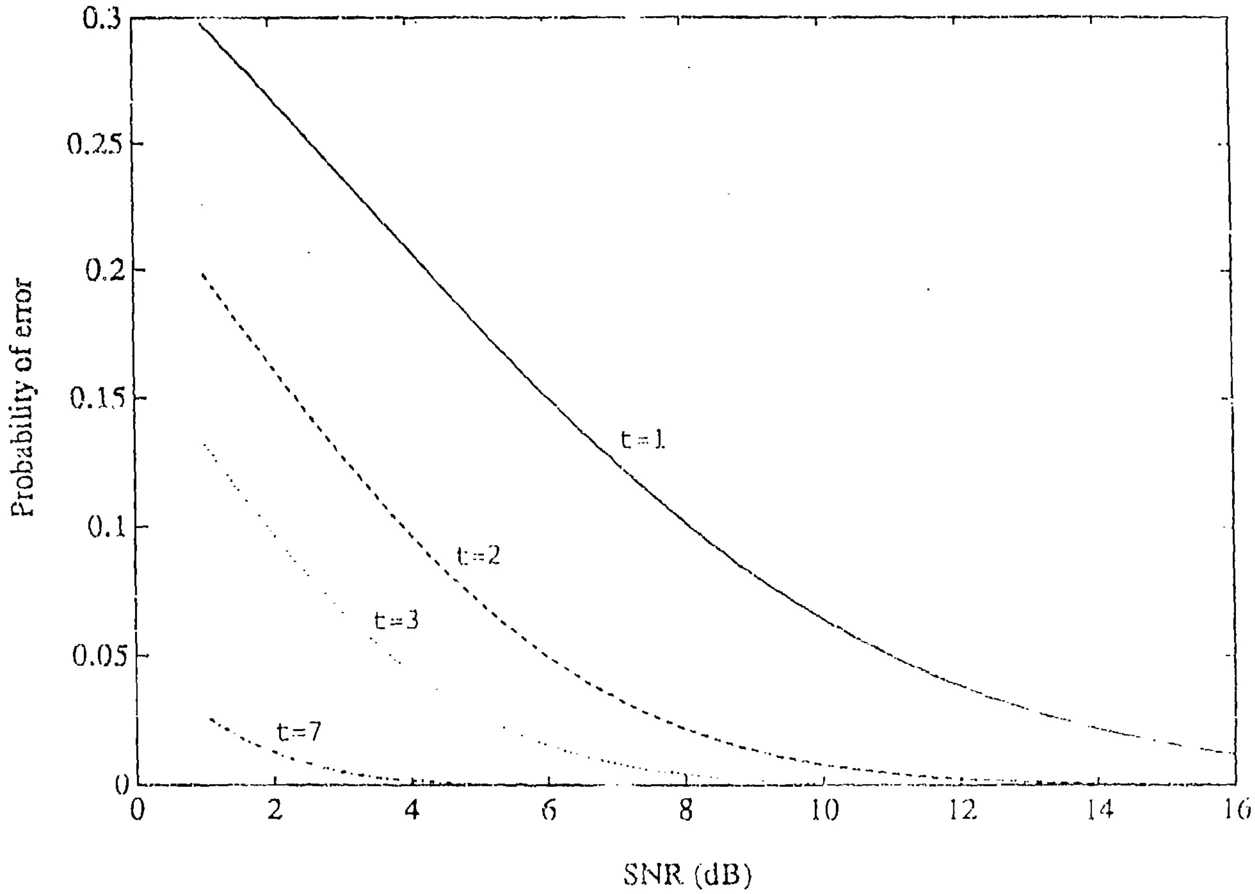

Figure 4.7: Probability of error for the system with peer communication , OR rule

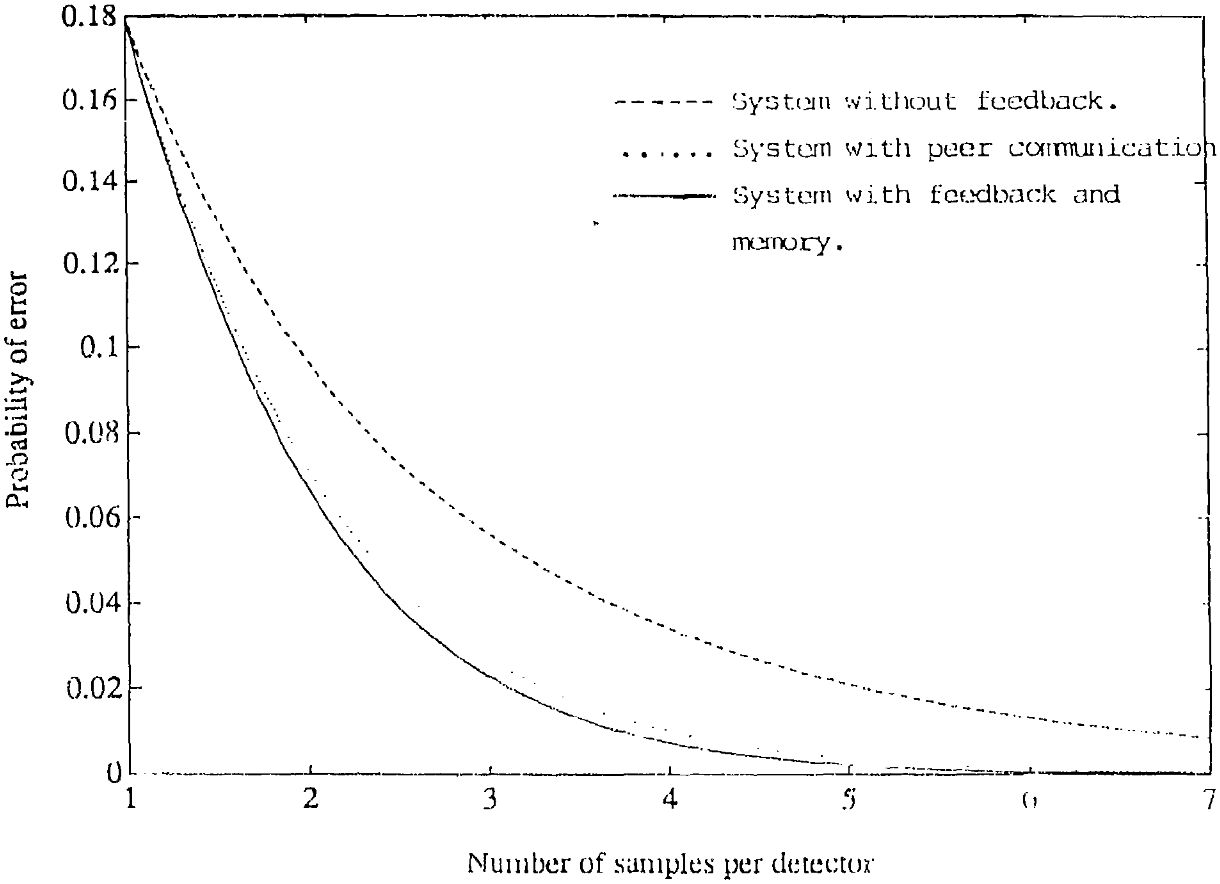

Figure 4.8: Probability of error for various decentralized detection systems, OR rule

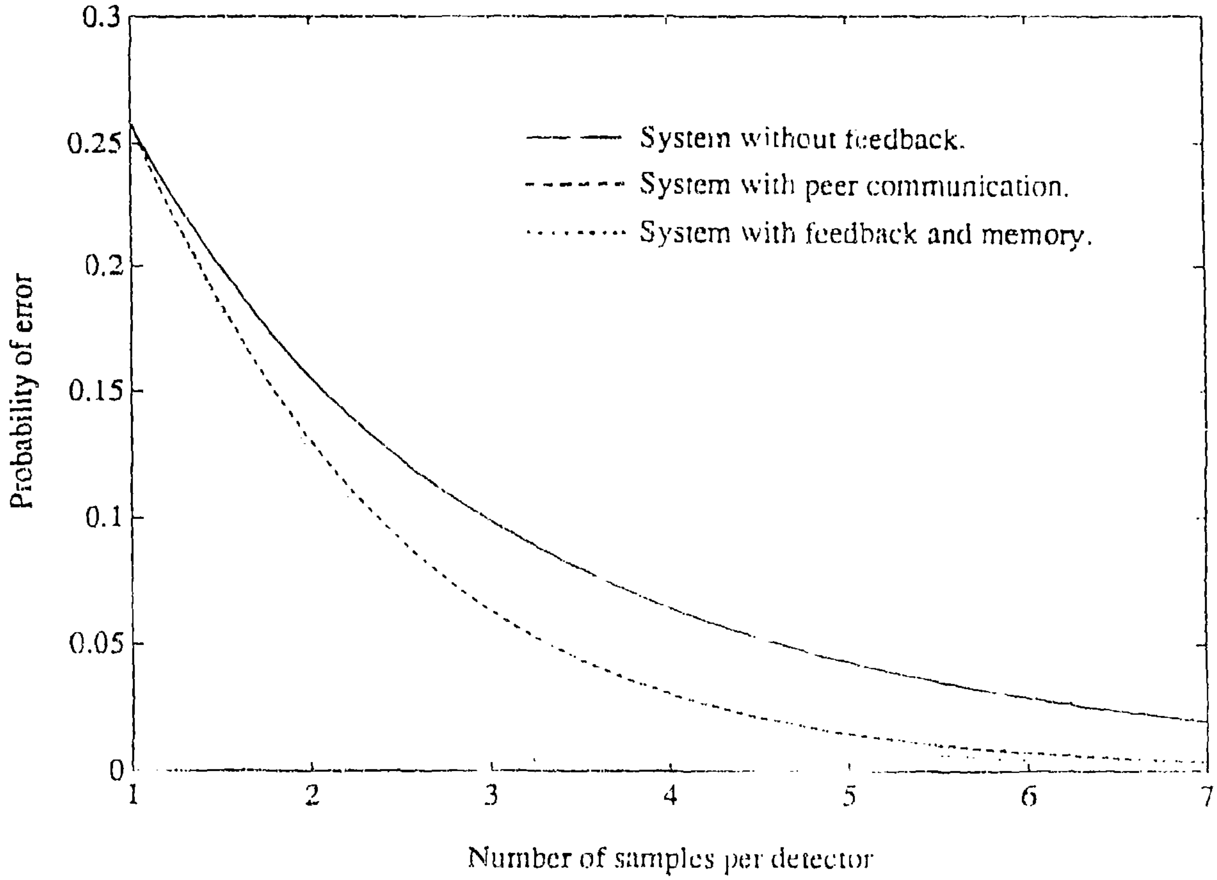

Figure 4.10: Probability of error for various decentralized detection systems. AND rule

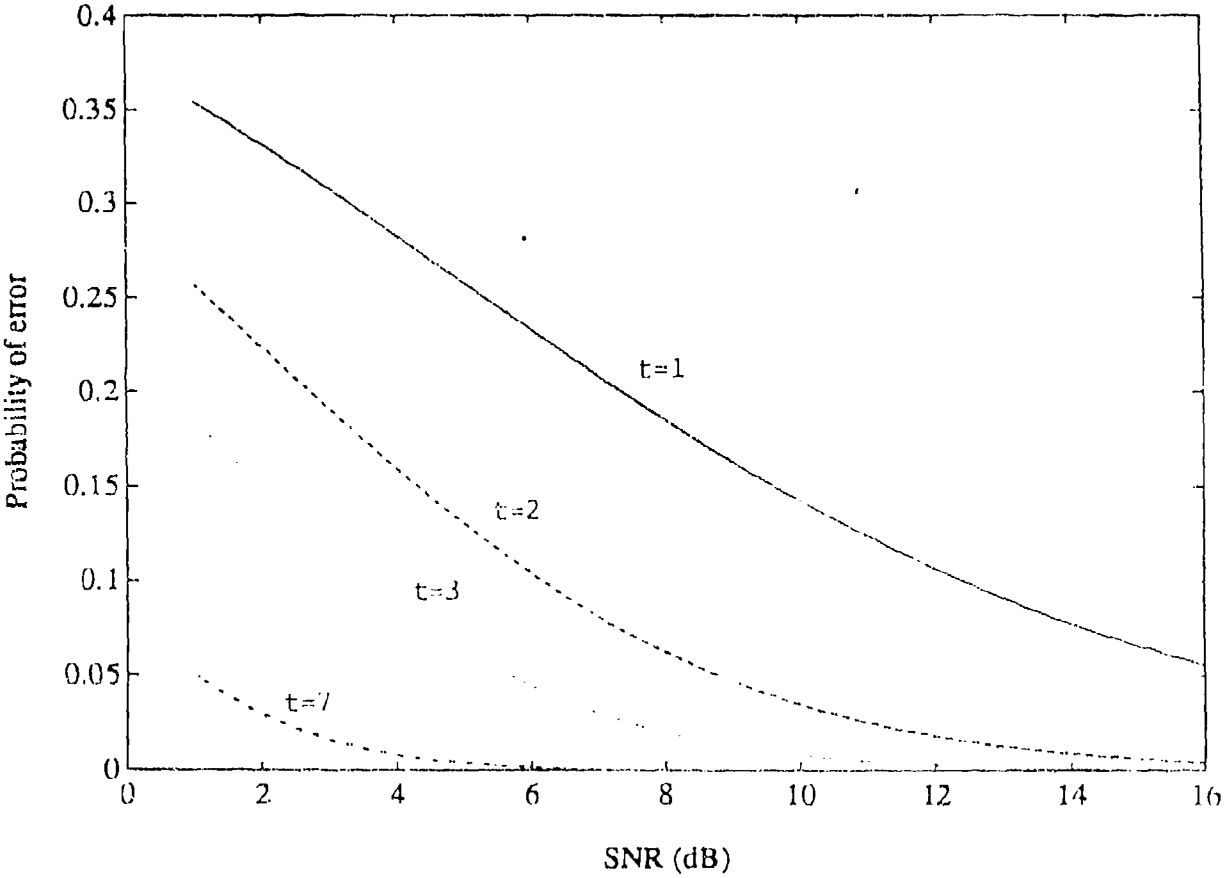

Figure 4.9: Probability of error for the system with peer communication, AND rule

interconnection between detectors in a decentralized detection system by a communication matrix. Based on this representation, we have derived a general PBPO decision rule at any detector of a decentralized detection system with any configuration. It was demonstrated that the unified approach can be used to obtain results from earlier chapters as well as results available in the literature. A new topological structure namely a decentralized detection system with peer communication was considered in detail and its performance was evaluated. Numerical results were also obtained for the case of two detectors and a fusion center. Our results in this chapter provide a versatile tool for the design and analysis of decentralized detection systems.

Chapter 5

Summary And Suggestions For Future research

5.1 Summary

In this dissertation, we have considered the binary hypothesis testing problem for a decentralized detection system with feedback consisting of n local detectors. Using the Bayesian formulation, we derived the local and the global decision rules. An expression for the system probability of error was also derived. It was shown that as the number of observation samples increased, the system probability of error decreased at a slower rate than that of a conventional decentralized detection system. The FSS problem was investigated where the stopping time was known a priori. Local and global threshold equations were derived and shown to be coupled spatially and temporally. The single detector system with feedback was studied. The decision rule of the single detector was derived for the FSS problem. It was shown that the single detector system with feedback corresponds to a serial system consisting of N detectors. The decision rule at time step t of the single detector

system with feedback was shown to be the same as the decision rule of the n^{th} detector of a serial network. Hence, results of the decentralized detection system with feedback could be extended to networks with blocks of detectors in tandem.

Next, a decentralized detection system with feedback incorporating memory at the local detectors was investigated. Using the PBPO solution methodology, local and global decision rules were derived. The system probability of error for this system was shown to be at least as good as the conventional decentralized detection system without feedback. Asymptotic behavior of the system probability of error was considered. It was shown that as the number of observations goes to infinity the system probabiliy of error goes to zero. Due to the feedback links, an increase in data transmission is exhibited. Two protocols were proposed and studied for the reduction of data transmission. It was shown that the average number of decision transmissions goes to zero as the number of samples goes to infinity. For a decentralized detection system with feedback and memory, the system probability of error and the average number of decision transmission were considerably better than that of the corresponding system without memory.

Finally, we presented the definition of the communication structure of decentralized detection systems. Then, using the Bayesian formulation and the PBPO solution methodology, the FSS problem was solved for a decentralized detection system with an arbitrary configuration. We derived the decision rules for a general decentralized detection system. Using these decision rules, we verified various results from the literature as well as the decentralized detection system with feedback. Using our new definition alongwith our decision rule design approach, we established a unified approach to the design and study of decentralized detection systems.

There are two major contributions of this dissertation. The first one is the

demonstration of the fact that the performance of a decentralized detection system can be improved by the use of feedback. This improvement is achieved at the expense of increased communication. The other major contribution is a unified representation of decentralized detection system with any topology along with an approach to obtain the PBPO decision rules at any detector of the decentralized detection system.

5.2 Suggestions For Future Research

Throughout this dissertation, we have assumed that the observations at the local detectors are statistically independent and identically distributed. In practice, however, spatial and temporal dependence of observations can be expected. Therefore, a fruitful area of research is to optimize the decentralized detection system with feedback under the appropriate dependent observation models. Another possibility is to investigate the non parametric problem in a decentralized detection system with feedback. Proper decision rules must be developed for this case.

The design of optimum decentralized detection systems is computationally quite intensive. It usually involves solutions of coupled nonlinear equations to determine the thresholds. Computationally efficient approaches for the design of optimum (or near-optimum) decentralized detection systems should be developed.

References

[1] H. L. Van Trees, Detection Estimation, and Modulation theory, Vol. I, J. Wiley, New York, 1968.

[2] A. Wald, Sequential Analysis, Wiley, New York, 1947.

[3] A. D. Whalen, Detection of Signals in Noise, Academic press, 1971.

[4] R. R. Tenney and N. R. Sandell, Jr., " Detection with distributed sensors," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-17, pp. 501-510, July 1981.

[5] F. A. Sadjadi, " Hypothesis testing in distributed environment," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-22, pp. 134-137, March 1986.

[6] Z. Chair and P. K. Varshney, " Optimum data fusion in multiple sensor detection system," *IEEE Trans. Aerospace and Electronic Systems*, Vol. AES-22, pp. 98-101, Jan. 1986.

[7] I. Y. Hoballah and P. K. Varshney, " Distributed Bayesian signal detection," *IEEE Trans. on Information Theory*, Vol. IT-35, pp. 988-944, Sep. 1989.

[8] A. R. Reibman and L. W. Nolte, " Optimal detection and performance of distributed sensor systems," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-23, pp. 24-30, Jan. 1987.

[9] A. R. Reibman and L. W. Nolte, " Design and performance comparison of distributed detection networks," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-23, pp. 789-797, Nov. 1987.

[10] G. Lauer and N. R. Sandell, Jr., " Distributed detection for known signals in correlated noise," TP-131, ALPHATECH Inc. Burlington, MA, Mar. 1982.

[11] L. K. Ekchian and R. R. Tenney, " Detection Networks," Proceedings of the 21st IEEE Conf. on Decision and Control, Orlando, pp. 686-691, Fl. 1982.

[12] H. J. Kushner and A. Pacut, " A simulation study of a decentralized detection problem," *IEEE Trans. on Automatic Control*, Vol. AC-27, pp. 1116-1119, Oct. 1982.

[13] D. Teneketzis, " The Decentralized Wald problem," Proceedings of the American Control Conference, San Fransisco, CA., 1983.

[14] D. Teneketzis and P. Varaiya, " The Decentralized quickest detection problem," *IEEE Trans. on Automatic Control*, Vol. AC-29, No. 7, pp. 641-644, July 1984.

[15] R. Srinivasan, " A theory of distributed detection," *Signal Processing*, Vol. 11, No. 4, pp. 319-327, Dec. 1986.

[16] H. M. Taylor and S. Karlin, An Introduction to Stochastic Modeling, Academic Press, Orlando, Fl. 1984.

[17] R. Viswanathan and S. C. A. Thomopoulos, " Optimal serial distributed decision fusion," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-24, No. 4, pp. 366-375, July 1988.

[18] M. M. Al-Ibrahim and P. K. Varshney, " A Simple multi-sensor sequential detection procedure," Proc. of the 27th IEEE Conf. on Decison and Control, pp. 2479-2483, Austin, TX. 1988.

[19] J. D. Papastavrou and M. Athans, " On optimal distributed decision architectures in a hypothesis testing environment," TR-LIDS-P-1928, M.I.T., Cambridge, Ma., Jan. 1990.

[20] N. T. Gaarder, J. K. Wolf, "The capacity region of a multiple-access discrete memoryless channel can increase with feedback," *IEEE Trans. on Information Theory*, Vol. IT-21, pp. 100-102, Jan. 1975.

[21] J. N. Tsitsiklis, " Decentralized detection," TR-LIDS-P-1913, M.I.T., Cambridge, Ma., Sep. 1989.

[22] Y. C. Ho, " Team decision theory and information structures in optimal control problems Part-1," *IEEE Trans. on Automatic control*, Vol. AC-17, No. 1, pp. 15-22, Feb. 1972.

[23] Y. C. Ho, " Team decision theory and information structures ," Proceedings of the IEEE, Vol. 68, No. 6, pp. 644-654, June 1980.

[24] J. N. Tsitsiklis and M. Athans, " Convergence and asymptotic agreement

in distributed decision problems," *IEEE Trans. on Automatic Control*, Vol. AC-29, No. 1, pp. 42-50, Jan. 1984.

[25] R. Radner, " Team decision problems," *Annals of Mathematical Statistics*, Vol. 33, No. 3, pp. 857-881, 1962.

[26] R. Srinivasan, " Distributed detection with decision feedback," IEEE Int. Symp. on Information Theory, San Diego, CA., 1990.

[27] S. Alhakeem, R. Srinivasan, and P. K. Varshney, " Decision agreement and link usage in distributed detection systems with feedback," IEEE Int. Symp. on Information Theory, San Diego, CA., 1990.

[28] J. V. DiFranco and W. L. Rubin, Radar Detection, Englewood Cliffs, N. J. Prentice-Hall, 1968.

[29] J. N. Tsitsiklis and M. Athans, " On the complexity of decentralized decision making problems," *IEEE Trans. on Automatic Control*, Vol. AC-30, No. 5, pp. 440-446, May 1985.

[30] M. I. Skolnick, Radar Handbook, McGraw-Hill , 1970.

[31] R. G. Gallager, Information Theory and Reliable Communication, J. Wiley and Sons, Inc., New York, 1968.

[32] R. R. Tenney and N. R. Sandell, Jr., "Strategies for distributed decision making," *IEEE Trans. on Systems, Man, and Cybernetics*, SMC-11, pp. 501-509, 1981.

[33] H. R. Hashemi and I. B. Rhodes, " Decentralized dynamic decision making," Proceeding of the 26th Conference on Decision and Control, Los Angeles, CA., Dec. 1987. pp. 1836-1842.

[34] H. R. Hashemi and I. B. Rhodes, " Distributed sequential detection," *IEEE Trans. on Information Theory*, Vol. IT-35, pp. 509-520, May 1989.

[35] R. E. Blahut, Pinciples And Practice Of Information Theory, J. Wiley, New York, 1987.

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